Extraction of V_{us} from Kaon Decays

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Determination of V_{us}

|Vus| can be extracted from K → πlν via

$$|V_{us}|.f_{+}^{K\pi}(0)=\sqrt{\frac{128\pi^{3}\Gamma(Kl3(\gamma))}{C^{2}G_{F}^{2}M_{K}^{5}S_{EW}I_{K}}}$$

Where:

Sew = 1.0232 – short distance enhancement factor,

$I_K(f_{+,0}^{K\pi}(t))$ - phase space integral, $C=\begin{cases}1 & K_{e3}^{0}\\ 1/\sqrt{2} & K_{e3}^{+}\end{cases}$

Long distance radiative corrections are included in I_k and $f_+(0)$

The physical quantity

$$\Gamma(\mathrm{K}_{e3(\gamma)})=\Gamma(\mathrm{K}_{e3})+\Gamma(\mathrm{K}_{e3\gamma})+\ldots$$

where the radiative corrections with virtual and real photos are taken into account Is well defined, calculable and measurable!

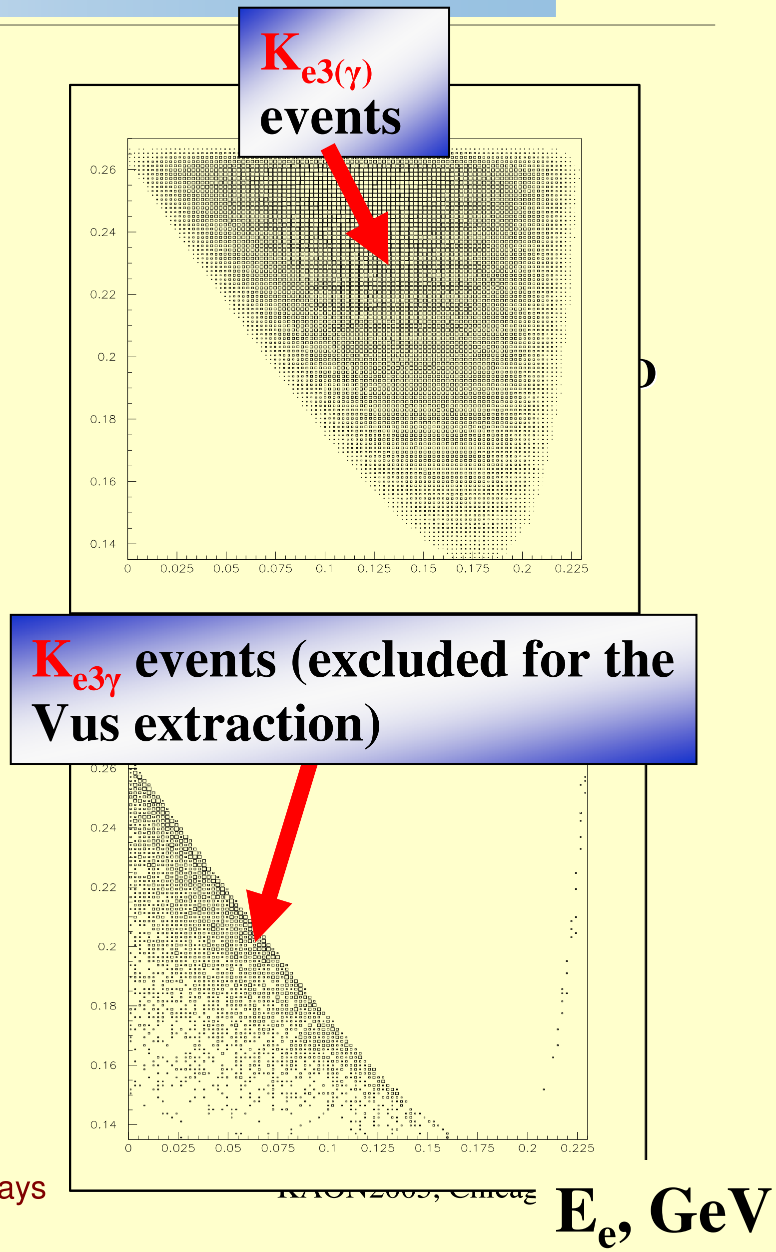
Determination of V_{us}

Prescription

- Accept all photon energies
- Accept all angles between pion and positron
- Accept only pion and positron energies within the original 3-body Dalitz plot.
- Inclusive rate obtained by integrating over the original domain

Experimentally – inclusive measurement of Br(Ke3). For determination of Vus, the corresponding Br(Ke3) should be inside the Dalitz plot. Corrections are:

C_K=0.49% for K^0 and C_K=0.50% for K^+



Determination of V_{us}

We can use three different parameterizations of the formfactors

Linear

$$f_{+,0}^{(o)}(t) = f_{+}^{(o)}(0)[1 + \lambda_{+,0} \frac{t}{m_{\pi^{\pm}}^{2}}]$$

Quadratic

$$f_{+,0}^{(o)}(t) = f_{+}^{(o)}(0)[1 + \lambda_{+,0} \frac{t}{m_{\pi^{\pm}}^{2}} + \lambda'_{+,0} \frac{t^{2}}{m_{\pi^{\pm}}^{4}}]$$

Pole

$$f_{+,0}^{(o)}(t) = f_{+}^{(o)}(0) \frac{m_{+,0}^{2}}{m_{+,0}^{2} - t}$$

Input for calculation of V_{us}

Experimental data

- ✓ Br(Ke3)
- ✓ Mean life times of K^0_L, K^0_S, K^+
- ✓ Linear and quadratic slopes of $f_+(t)$ – λ_+, λ_0 and λ_+, λ_+', λ_0

➢ Theoretical input – $f_+^{K\pi}(0)$

Experimental data

To have comparable results

Experimental data should be treated in the same way

-Inclusive measurement of the Br(Ke3)

-Correct account for radiative corrections, including real photons

Two classes of data on measurement of Br(Ke3)

Old data – actually what is included in PDG 2004

New data – published or reported in 2003, 2004 and 2005

Experimental data

The careful investigation of old experimental data on measurement of Br(Ke3) leads to the definite conclusion that due to different reasons they are not enough accurate and are not suitable for extraction of Vus matrix element

In what follows we will use only the new high statistics experimental data on measurement of Br(Ke3)

$K^0_L \to \pi e \nu$ – NA48 **result**

Experimental result

$Br(K^0_L\ e3)/Br(2tr) = 0.4978 \pm 0.0035$

To determine $Br(K^0_L \to \pi e \nu)$ we need $Br(K^0_L \to 3\pi^0)$

PDG04: $Br(K^0_L \to 3\pi^0) = 0.2105 \pm 0.0028$

KTeV $Br(K^0_L \to 3\pi^0) = 0.1945 \pm 0.0018$

Average according PDG prescription

$Br(K^0_L \to 3\pi^0) = 0.1992 \pm 0.0070$

$Br(K^0_L\ e3) = 0.4010 \pm 0.0028_{exp} \pm 0.0035_{norm}$

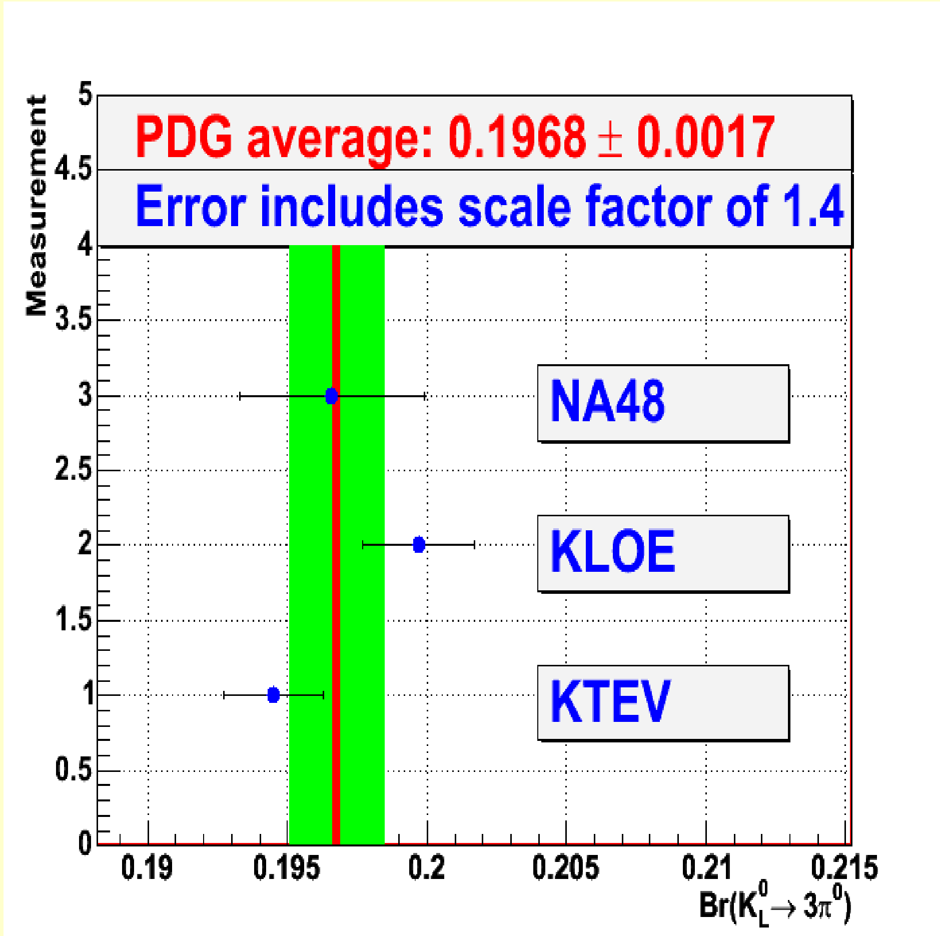
Results for Br($K^0_L \to 3\pi^0$)

Taking into account the KTeV, NA48 and KLOE results we obtain

Br($K_L \to \pi^0\pi^0\pi^0$) = 0.1968 ± 0.0017

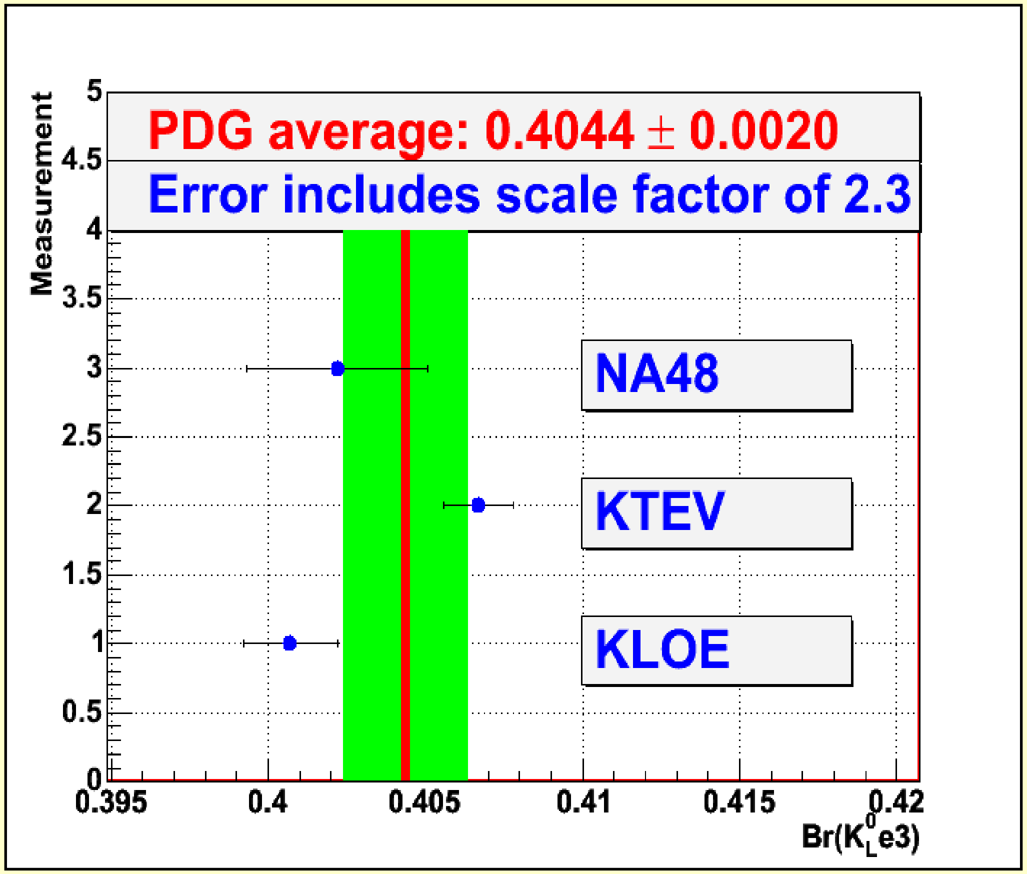
Than the NA48 result changes to

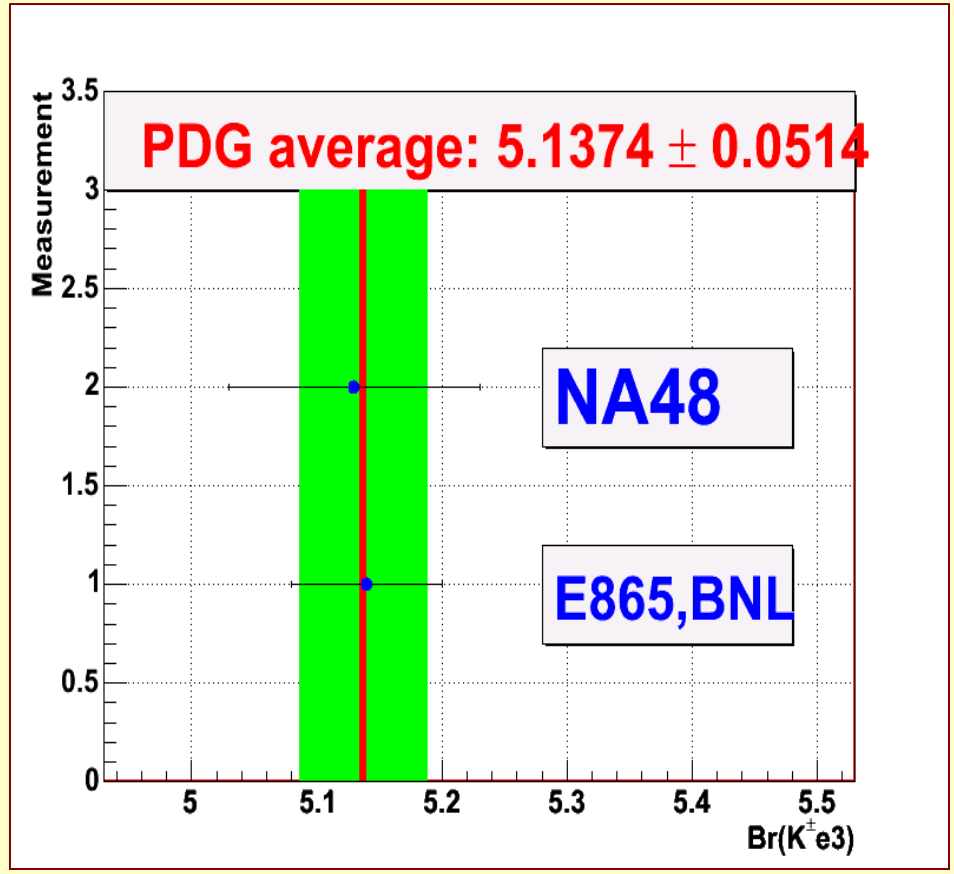
Br(K^0_L e3) = 0.4022 ± 0.0029



Determination of V_{us}

Br(Ke3)





$Br(K^0_L\ e3) = 0.4044 \pm 0.0020$

$Br(K^\pm\ e3) = (5.137 \pm 0.051)\%$

Determination of V_{us}

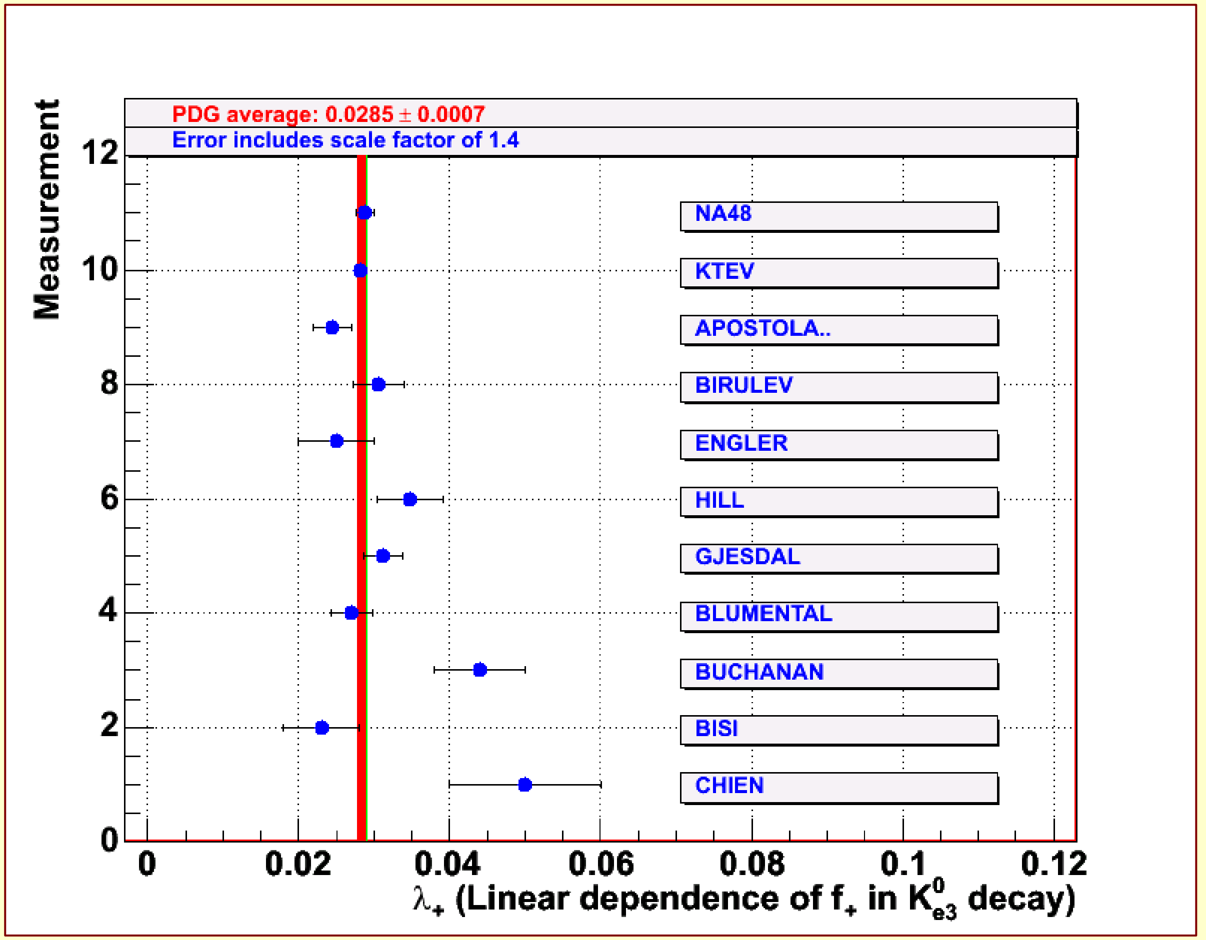
Slope of the $f_+(t)$ – K_L^0e3

Linear approximation

$\lambda_+ = 0.0285 \pm 0.0007$

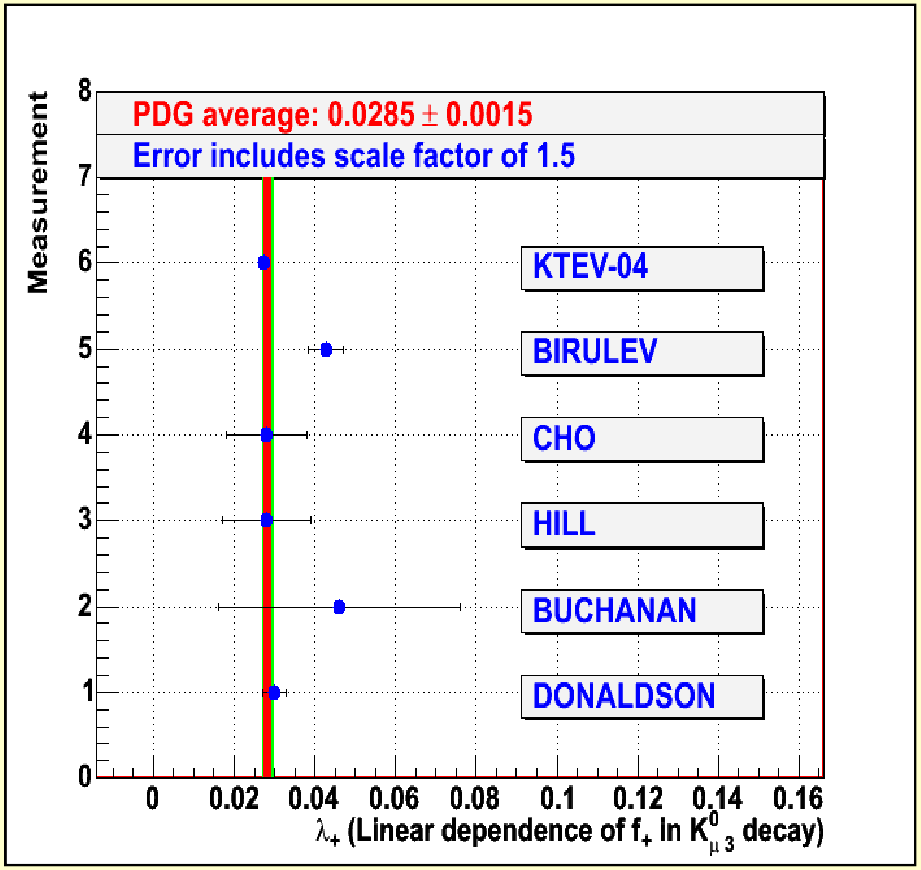
Quadratic approximation
KTeV result

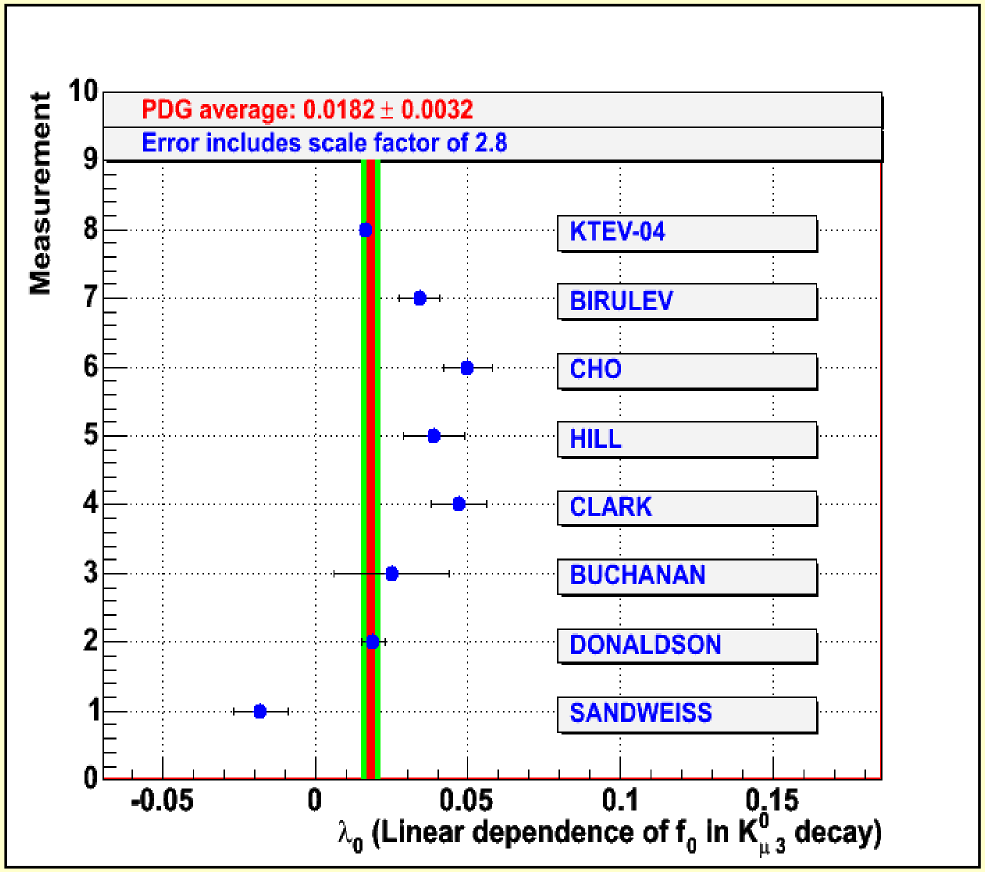
$\lambda_+ = 0.02167 \pm 0.00199$
$\lambda_+' = 0.00144 \pm 0.00039$



Determination of V_{us}

Slope of the $f_+(t)$ – $K^0_{\mu 3}$





$\lambda_+ = 0.0285 \pm 0.0015$

$\lambda_0 = 0.0182 \pm 0.0032$

Determination of V_{us}

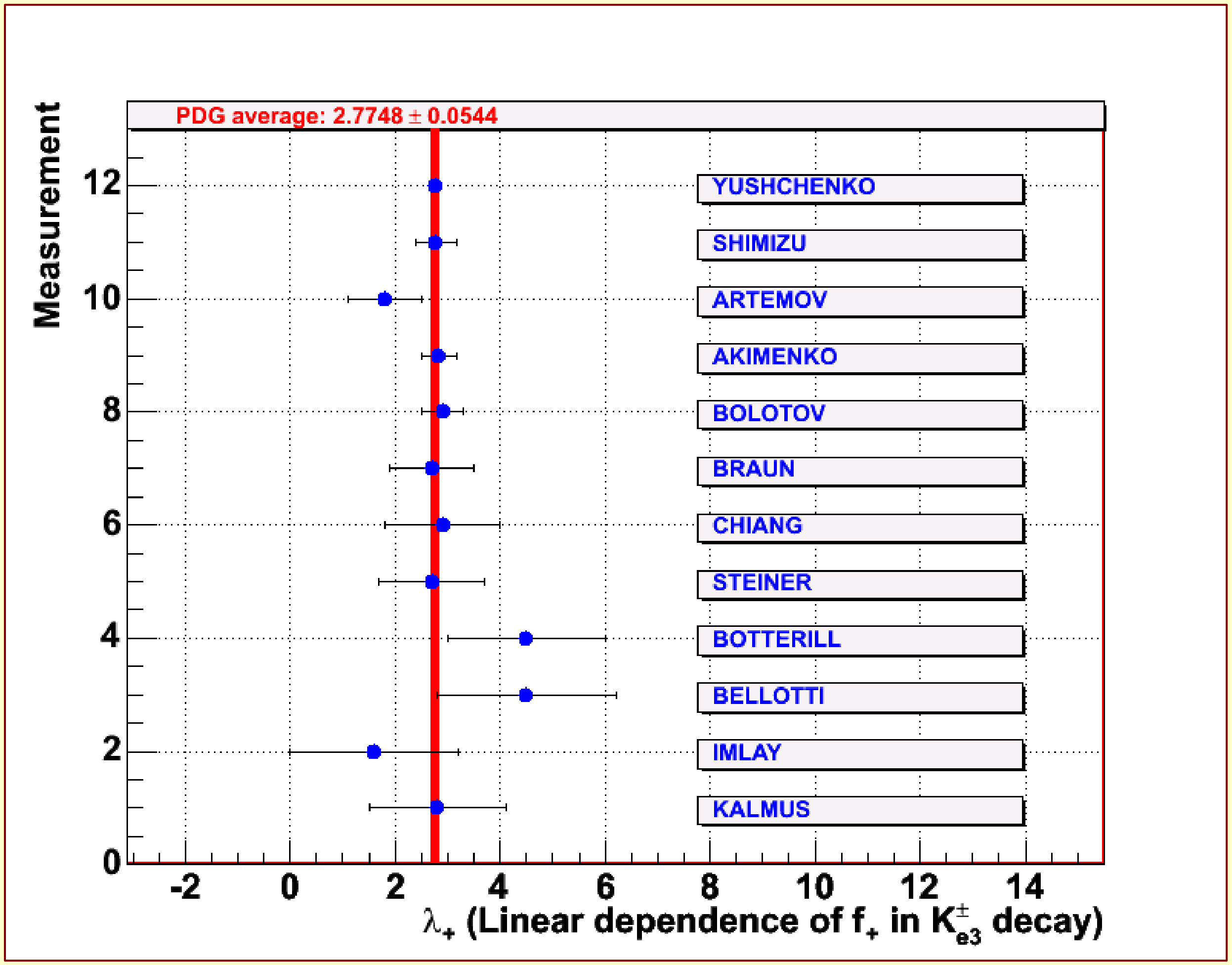
Slope of the $f_+(t)$ – $K^{\pm}$

Linear approximation

$\lambda_+ = 0.0277 \pm 0.0005$

Quadratic approximation
ISTRA+ result

$\lambda_+ = 0.02324 \pm 0.00155$
$\lambda_+' = 0.00084 \pm 0.00041$

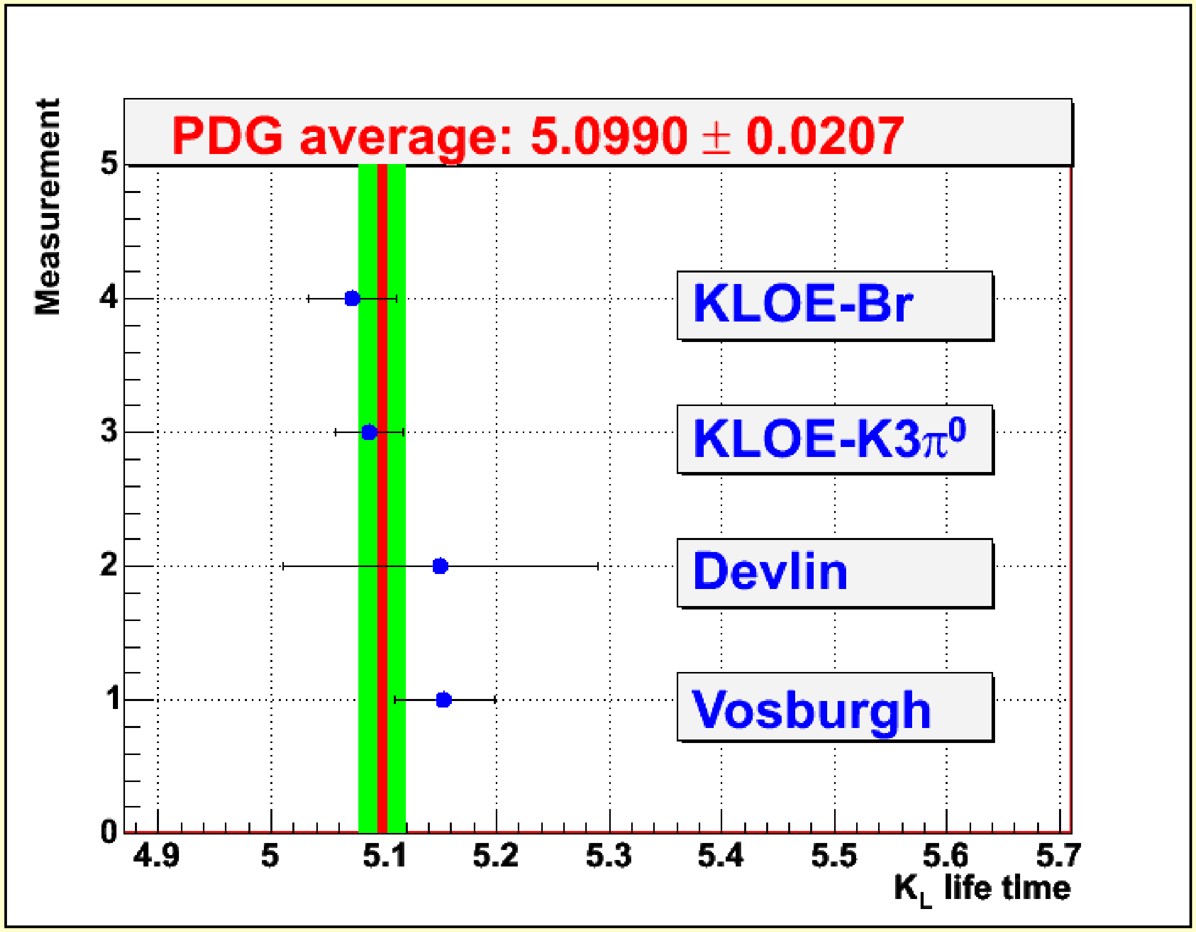


Mean life time

K_L^0

New KLOE results

$\tau = (5.099 \pm 0.021).10^{-8}$s



$K^{\pm}$

$\tau = (1.2385 \pm 0.0025).10^{-8}$s

Determination of V_{us}

Calculation of $f_+^{K\pi}(0)$

Let us represent $f_+(t)$ in the following form:

$$f_+ = \tilde{f}_+(t) + \hat{f}_+(t)$$

$\tilde{f}_+(p^4)$ $\quad$ $\tilde{f}_+(p^6)$

QCD effects to $O(p^6)$
EM contribution to $O(e^2p^2)$
EM contraterms relevant to π^0 - η mixing

local effects of virtual photon exchange of order $O(e^2p^2)$

Determination of V_{us}

Calculation of $f_+^{K\pi}(0)$

Calculation of $f_+(0)$ to $O(p^4)$ - Gasser & Leutwyler
First calculation to $O(p^6)$ – Leutwyler & Roos
QCD + isospin breaking

$$\tilde{f}_+^{K^0\pi^-}(0) = 0.961 \pm 0.008$$

$$\tilde{f}_+^{K^+\pi^0}(0) = 0.982 \pm 0.008$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -0.016 \pm 0.008$$

Bijnens & Talavera

$$\tilde{f}_+^{K\pi}(0) = 0.976 \pm 0.010$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -8\left(\frac{M_K^2 - M_\pi^2}{F_\pi^2}\right)[C_{12}^r(\mu) + C_{34}^r(\mu)] + \Delta_{loops}(\mu)$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6}^{local} = -0.016 \pm 0.008$$

$$\Delta_{loops}(M_\rho) = 0.0146 \pm 0.0064$$

Determination of V_{us}

Calculation of $f_+^{K\pi}(0)$

Quenched lattice calculations – Becirevic at al.

$$\tilde{f}_+^{K^0\pi^-}(0) = 0.960 \pm 0.009$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -0.017 \pm 0.008$$

Cirigliano, Neufeld and Pichl

Calculation using χPT with virtual photons and leptons

- **Isospin breaking by the quark masses up to $O((m_u-m_d)p^2)$**
- **Isospin conserving contribution from SU(3) breaking $O(p^6)$**
- **Electromagnetic effects up to $O(e^2p^2)$**

Determination of V_{us}

Calculation of $f_+^{K\pi}(0)$

To extract Vus we have used the following values

	LO + NLO QCD	EM. radiative corrections	NNLO QCD	total
K^0 K^+	0.97699±0.00002 1.0002±0.0022	0.0046±0.0008 0.0032±0.0016	-0.001±0.010 -0.001±0.010 0.007±0.012	0.981±0.010 1.002±0.010
K^0 K^+	0.97699±0.00002 1.0002±0.0022	0.0046±0.0008 0.0032±0.0016	-0.017±0.009 -0.017±0.009	0.965±0.009 0.986±0.010

The main uncertainty (~1%) comes from $O(p^6)$ contribution

Determination of V_{us}

Results – K^0

$$f_{+}^{K^0\pi^+}(0) = 0.965 \pm 0.009$$

Linear approximation of $f_+(0)$

Experiment	Br	$V_{us}f_+(0)$	V_{us}
PDG	0.3881±0.0027	0.2125±0.0009	0.2202±0.0022
NA48	0.4022±0.0029	0.2163±0.0009	0.2241±0.0022
KTeV	0.4067±0.0011	0.2175±0.0006	0.2254±0.0021
KLOE – K_L	0.4007±0.0015	0.2159±0.0006	0.2237±0.0021
KLOE – K_S	$(7.09\pm0.11).10^{-4}$	0.2165±0.0017	0.2244±0.0021
KTeV – $K_{\mu3}$	0.2701±0.0009	0.2177±0.0007	0.2256±0.0021
KLOE – $K_{\mu3}$	0.2698±0.0015	0.2176±0.0009	0.2255±0.0021
Average K_L	0.4044±0.0020	0.2169±0.0007	0.2248±0.0024

Determination of V_{us}

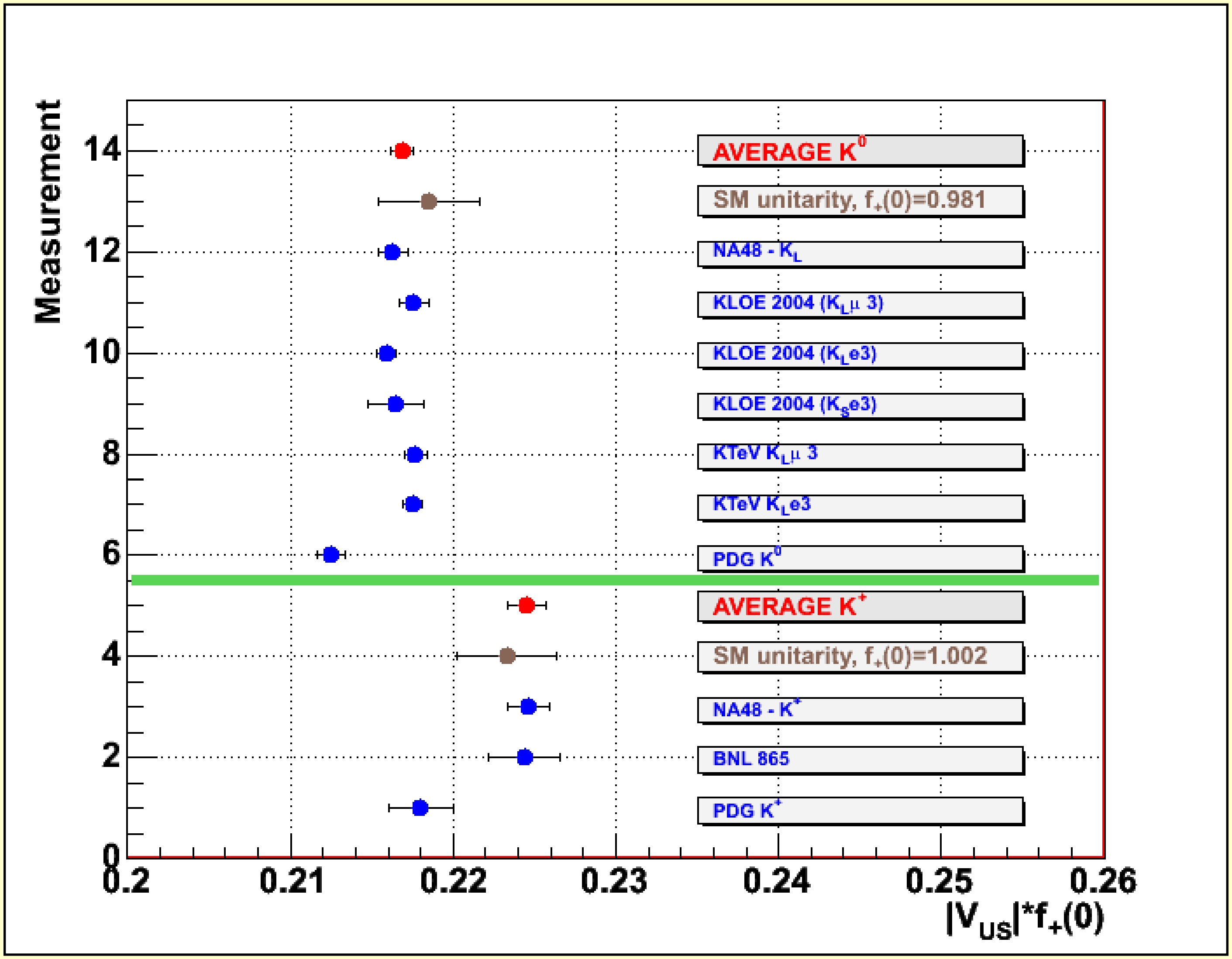
Results – K^+

Linear approximation of $f_+(0)$

$$f_+^{K^+\pi^0}(0) = 0.986 \pm 0.010$$

Experiment	Br [%]	$V_{us}f_+(0)$	V_{us}
PDG	4.84±0.09	0.2180±0.0020	0.2176±0.0030
NA48	5.14±0.06	0.2246±0.0013	0.2278±0.0026
E865	5.13±0.10	0.2244±0.0022	0.2276±0.0031
Average	5.137±0.051	0.2245±0.0012	0.2277±0.0025

Determination of V_{us}



Consistency of K_L and K_{ch} data

Ratio of $f_+(0)$ for K_L and K^+ can be measured

$$R = f_+^{K^0\pi^+}(0) / f_+^{K^+\pi^0}(0)$$

Its calculation is free from many of the theoretical uncertainties

$$R^{th} = 1.022 \pm 0.003 - 16\pi\alpha X_1$$

$$1.017 \le R^{th} \le 1.027$$

From the averaged K_L and K^+ data we obtain

$$R^{\exp} = 1.035 \pm 0.006$$

In disagreement with theoretical predictions ~ 2σ

- **failure of the naïve dimensional analysis for X_1**
- **failure of chiral power counting**
- **wrong mean life times**

Determination of V_{us}

If we use for V_{us} determination

$$f_+^{K^0\pi^+}(0) = 0.981 \pm 0.010$$

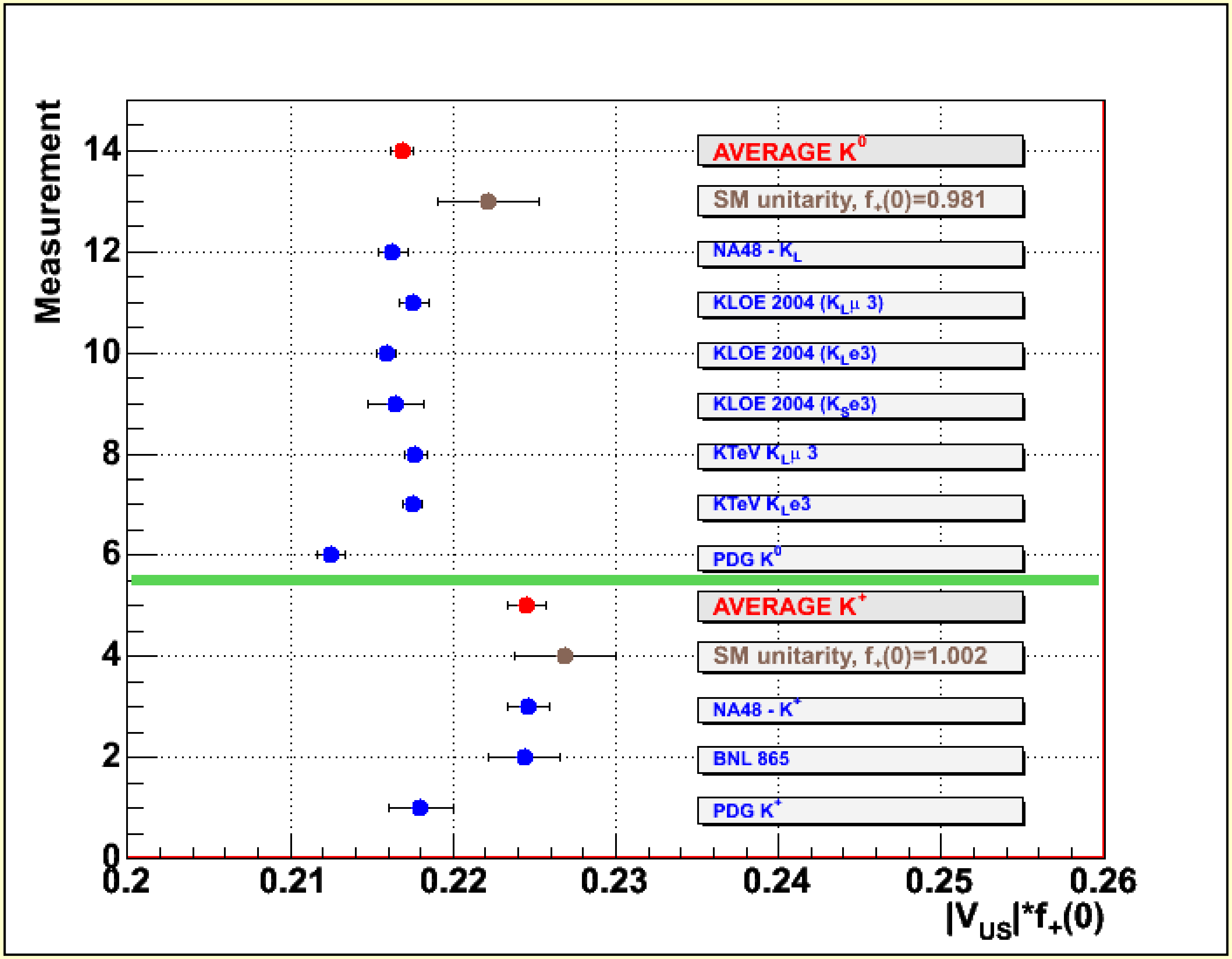
$$f_+^{K^+\pi^0}(0) = 1.002 \pm 0.010$$

$$|V_{us}|^{K^0\pi^+} = 0.2211 \pm 0.0022$$

$$|V_{us}|^{K^\pm\pi^0} = 0.2241 \pm 0.0025$$

The values of V_{us} are changed ~ 1.7σ
Calculation of $f_+(0)$ is the most important problem to be solved

Determination of V_{us}



Determination of V_{us}

Non linear approximation leads to

$$|V_{us}|.f_{+}^{K^0\pi^+}(0)=0.2179\pm0.0011$$

$$|V_{us}|.f_{+}^{K^\pm\pi^0}(0)=0.2253\pm0.0014$$

$$|V_{us}|^{K^0\pi^+}(0)=0.2258\pm0.0024$$

$$|V_{us}|^{K^\pm\pi^0}(0)=0.2285\pm0.0027$$

The values of $V_{us}f_{+}(0)$ are changed ~ 1σ and ~ 0.6σ
The values of Vus are changed ~ 0.4σ and ~ 0.3σ

In perfect agreement with unitarity of CKM matrix

Determination of $f_+(0)$

If we suppose that CKM matrix is unitary

$$|V_{us}| = 0.2265 \pm 0.0022$$

then we can determine the values of $f_+(0)$ using

$$|V_{us}|.f_+^{K^0\pi^+}(0) = 0.2169 \pm 0.0007$$

$$|V_{us}|.f_+^{K^\pm\pi^0}(0) = 0.2245 \pm 0.0012$$

$$f_+^{K^0\pi^+}(0) = 0.958 \pm 0.009$$

$$f_+^{K^+\pi^0}(0) = 0.991 \pm 0.010$$

The non linear approximation does not effects the result significantly

$$f_+^{K^0\pi^+}(0) = 0.962 \pm 0.008$$

$$f_+^{K^+\pi^0}(0) = 0.995 \pm 0.011$$

Conclusions- V_{us}

- The careful analysis of the existing data has shown
  - the old measurements are not suitable for determination of V_{us}
  - new measurement of all kaon branching fractions is desirable
  - new more precise measurements of Ke3 and Kμ3 form factors are needed
  - new measurement of the kaon mean life times will be welcome
  - KLOE and part of NA48 data are still preliminary
- Vus values obtained using average values of Br(Ke3)
  - Support the unitarity of CKM matrix
  - Strongly depend from the values of $f_+(0)$
  - More precise calculation of $O(p^6)$ contribution is required
- The experimental data for R are in disagreement with theoretical predictions
- Measured values of $f_+(0)$ (with unitary CKM matrix) cause questions to the theory

Old experimental data - $K^{\pm}$

Direct measurement of Br(Ke3)

- dominating experiment Chiang et al., Phys.Rev.D6, 1972, p.1254 accuracy ~ 2%

  Ke3 measurement is nor inclusive

  No radiative corrections

  The decays $\pi \rightarrow \mu$ are not taken into account

  The Dalitz decays of π^0 are not taken into account

In the PDG fit also contribute

- Br(Ke3)/Br(2π)

  -in the dominating experiment (~ 5% acc.) rad. corrections without real γ

- Br($K_{\mu3}$)/Br(K_{e3})

  K. Horie, Phys. Lett. B513, p. 311, 2001

  The measurement is not inclusive

  Ke3γ is considered as background

Determination of V_{us}

Old experimental data – K_L^0

There is no direct measurement of Br(Ke3)

In the PDG fit contribute

- $Br(K_{\mu3})/Br(K_{e3})$ – 4 experiments with good statistic
  - Two of them are perfect, both measure $Br(K_{\mu3})/Br(K_{e3}) = 0.662$ close to KTeV result
  - The other two – Hydrogen bubble chambers

    In this case separation of Ke3 and Kμ3 decays is extremely difficult

    ` 50% of the events are ambiguous – to separate complicated weighting procedure

    Their results shift $Br(K_{\mu3})/Br(K_{e3})$ to 0.697

    in strong disagreement with recent measurements
- The other contribution is from Kreutz, ZPHY C55, p.67, 1995 – the results from this experiment are in strong contradictions with recent measurements