

# Precise test of CKM matrix unitarity

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# Introduction

- CKM unitarity
- NA48 experimental setup
- Measurement of  $\text{Br}(K_L^0 e3)/\text{Br}(2\pi)$
- $\text{Br}(K_L^0 e3)$
- Measurement of  $\text{Br}(K_L^0 \rightarrow 3\pi^0)$
- Measurement of  $\text{Br}(K^\pm e3)$
- $K_L^0 e3$  form factors
- The radiative decay  $\text{Br}(K_L^0 e3\gamma)$
- Extraction of  $V_{us}$ 
  - ❖ Experimental data
  - ❖ Theoretical input
  - ❖ Results
  - ❖ Compatibility of  $K^0$  and  $K^{+/-}$  results
- Conclusions

# Introduction

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## CKM - Unitarity

# CKM Unitarity

Unitarity of CKM matrix leads to a number of relations between  $V_{ij}$

In particular for the first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Most precisely measured elements of CKM

PDG 2004 data

$|V_{ud}|$  - well determined from measurement of

super allowed nuclear  $\beta$ -decays

free neutron life time

$$|V_{ud}| = 0.9738 \pm 0.0005$$

$$|V_{ub}| = (3.67 \pm 0.47) \cdot 10^{-3} - (|V_{ub}|^2 \approx 10^{-5} \text{ negligible})$$

SM prediction

$$|V_{us}| = 0.2274 \pm 0.0021$$

# CKM Unitarity

Experimental value

$$|V_{us}| = 0.2200 \pm 0.0026$$

$$\Delta|V_{us}| = 0.0074 \pm 0.0033 \quad \sim 2.2 \sigma \text{ discrepancy}$$

To solve the problem – measurement with precision  $\sim 1\%$  (limited by theory)

Semileptonic decays  $K \rightarrow \pi e \nu$  best for determination of  $|V_{us}|$

The  $Ke3$  matrix element is parameterized by one form factor

$$M = C \frac{G_F}{\sqrt{2}} V_{us} l^\mu f_+^{(o)}(t) (p_K + p_\pi)_\mu$$

Vector current transition matrix element

$$f_+^{(o)}(t) (p_K + p_\pi)_\mu = \langle \pi | V_\mu^4 - iV_\mu^5 | K \rangle$$

$$f_+^{(o)}(t) = f_+^{(o)}(0) \left[ 1 + \lambda_+ \frac{t}{m_{\pi^\pm}^2} \right]$$

$\lambda_+$  experimentally measured

# CKM Unitarity

**Recent experimental data – evidence for non linear terms**

$$f_+^{(o)}(t) = f_+^{(o)}(0) \left[ 1 + \lambda_+ \frac{t}{m_{\pi^\pm}^2} + \lambda'_+ \frac{t^2}{m_{\pi^\pm}^4} \right]$$

Experimental data ~ 30 years old

Recent measurements -  $K^+e3$  (E865, 2003), NA48 and  
 $K^0e3$  - (KTeV), NA48, KLOE, prel  
are significantly above previous results.

Accuracy – better than 1%

NA48

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# NA48 Experiment

# NA48 Experiment

Situated at SPS accelerator in CERN, Geneva

Designed for measurement of direct CP-violation and rare Kaon decays

Simultaneous  $K_L$  and  $K_S$  beams with momentum (20-200) GeV/c

In 2003 NA48 beam line was upgraded to transport simultaneous  
60 GeV  $K^+$  and  $K^-$  beams

Beam spectrometer has been included in the experimental setup

Main goal – search for direct CP violation in  $K \rightarrow 3\pi$  decays

Precise measurement of semileptonic and radiative Kaon decays

Investigation of rare kaon decays



# NA48 experiment

## □ Main detector components

### ❖ Magnet spectrometer

- Two drift chambers before and two after spectrometer magnet
- Momentum resolution  $< 1\%$  for 20 GeV/c momentum

### ❖ Scintillator hodoscope (200 ps)

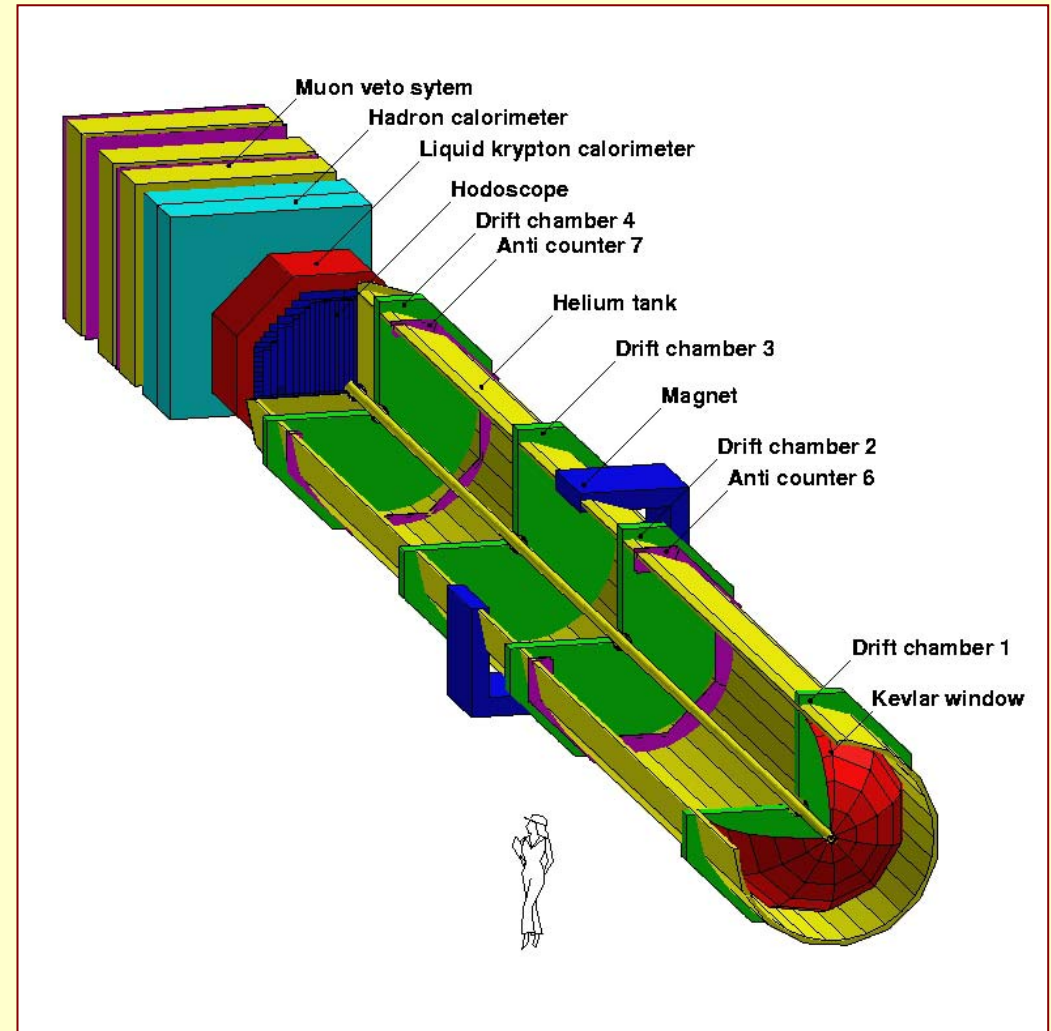
### ❖ Liquid Krypton Calorimeter

$$\frac{\delta E}{E} = \frac{3.2\%}{\sqrt{E[\text{GeV}]}} \oplus \frac{90\text{MeV}}{E} \oplus 0.42\%$$

### ❖ Hadron Calorimeter

### ❖ Muon Veto system

### ❖ Beams – $K_L^0, K_S^0, K^\pm$



## NA48 data

1997	$\epsilon'/\epsilon$ run	$K_L + K_S$
1998	$\epsilon'/\epsilon$ run	$K_L + K_S$
1999	$\epsilon'/\epsilon$ run $K_L + K_S$	$K_S$ Hi. Int.
2000	$K_L$ only <i>NO Spectrometer</i>	$K_S$ High Intensity
2001	$\epsilon'/\epsilon$ run $K_L + K_S$	$K_S$ High Int.
2002	$K_S$ High Intensity	
2003	$K^\pm$ High Intensity	
2004	$K^\pm$ High Intensity	

### NA48: 1997 – 2001

- Direct CP violation ( $\text{Re}(\epsilon'/\epsilon)$ )
- $K_L$  decays (e.g.  $K_{e3}^0 \rightarrow |V_{us}|$ )

### NA48/1: 2000, 2002

- High-intensity run for rare  $K_S$  decays.
- Hyperon decays ( $\Xi^0, \Lambda$ ) ( $\rightarrow |V_{us}|$ )
- Neutral  $K_S$  decays (2000)

### NA48/2: 2003 – 2004

- Search for direct CPV in  $K^\pm$  decays.
- Rare decays  
( $K_{e4}, K^+ \rightarrow \pi^+ e^+ e^-, K^+ \rightarrow \pi^+ \gamma \gamma, \dots$ )
- Semileptonic decays ( $K_{e3}^+ \rightarrow |V_{us}|$ )

## New NA48 results

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# Measurement of $\text{Br}(\text{Ke}3)$

## $K_L^0 \rightarrow \pi e \nu$

- ❖ Semileptonic  $K_L$  decays  $K_L^0 \rightarrow \pi l \nu$ 
  - Data from special minimum bias run 1999 with pure  $K_L^0$  beam
  - Very high statistics available – 80 million triggers taken
- ❖ General idea
  - Normalize to as many as possible channels
  - Data selection and analysis - as simple as possible
- ❖ Measure the ratio  $\text{Br}(K_L^0 \rightarrow e3)/\text{Br}(2\text{tr})$      $2\text{tr} =$  all  $K_L^0$  decays with two charged particles in the spectrometer
  - Normalization on
$$\text{Br}(2\text{tr}) = 1.0048 - \text{Br}(K_L^0 \rightarrow 3\pi^0)$$
is experimentally known

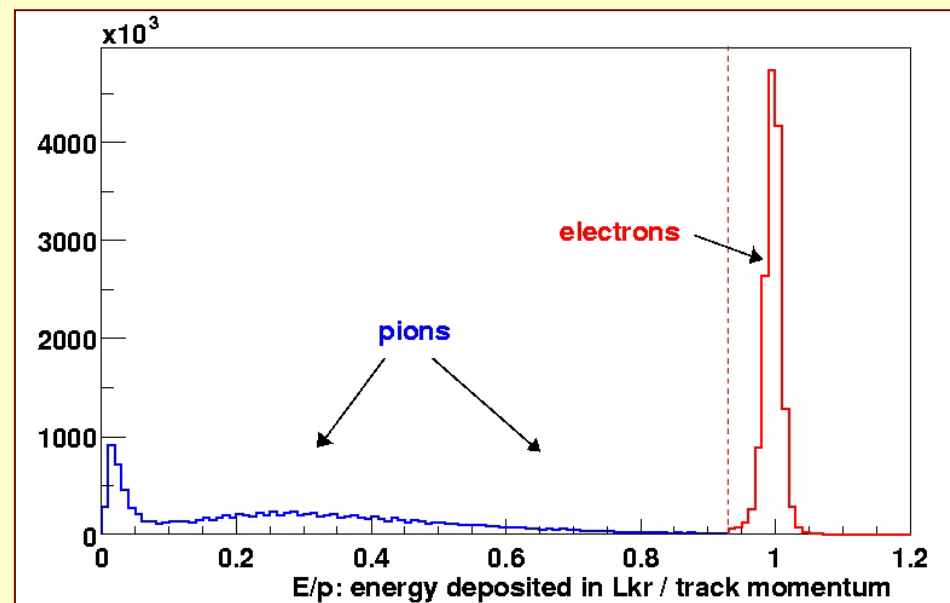
$$K_L^0 \rightarrow \pi e \nu$$

❖ Main selection criteria for 2 track sample

- Two tracks with opposite charges
- Decay vertex between 8 m and 33 m from final collimator
- Track separation in LKr > 25 cm
- Track momenta > 10 GeV
- $P_{\text{sum}} = P_1 + P_2 > 60 \text{ GeV}$

12.6 million 2 track events

- ❖  $K_L^0 \rightarrow \pi e \nu$  selection – the same but
- ❖ One of the tracks to be an electron
  - $E(\text{LKr})/p > 0.93$



# $K_L^0 \rightarrow \pi e \nu$

## ❖ Background to $K_{e3}$ sample

- BG from  $K\mu 3$  and  $K3\pi$  with  $\pi^\pm$  misidentified as  $e^\pm$
- Estimate the BG from  $K_{e3}$  data with identified  $e^\pm$  ( $E/p > 1$ )

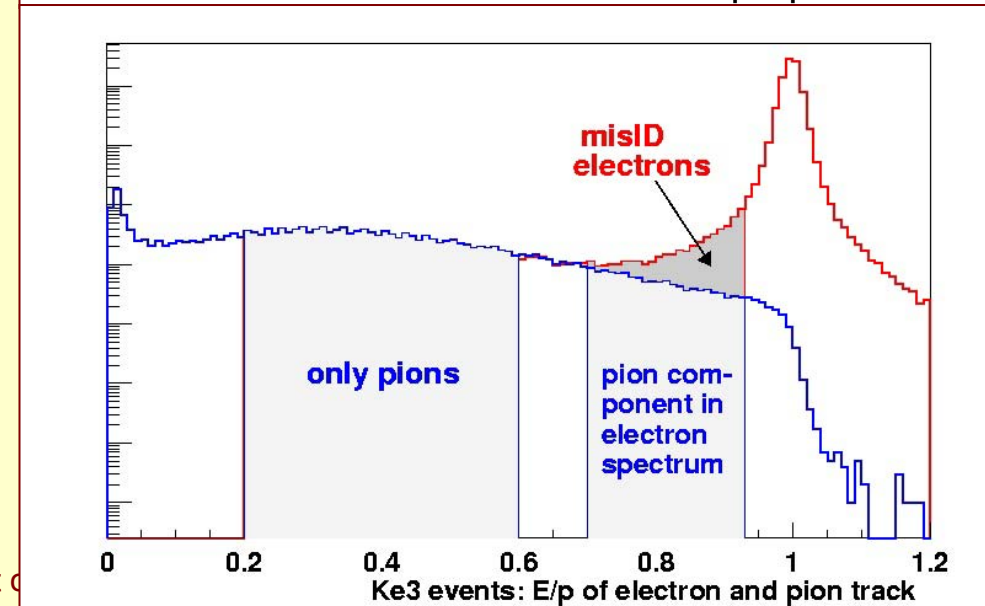
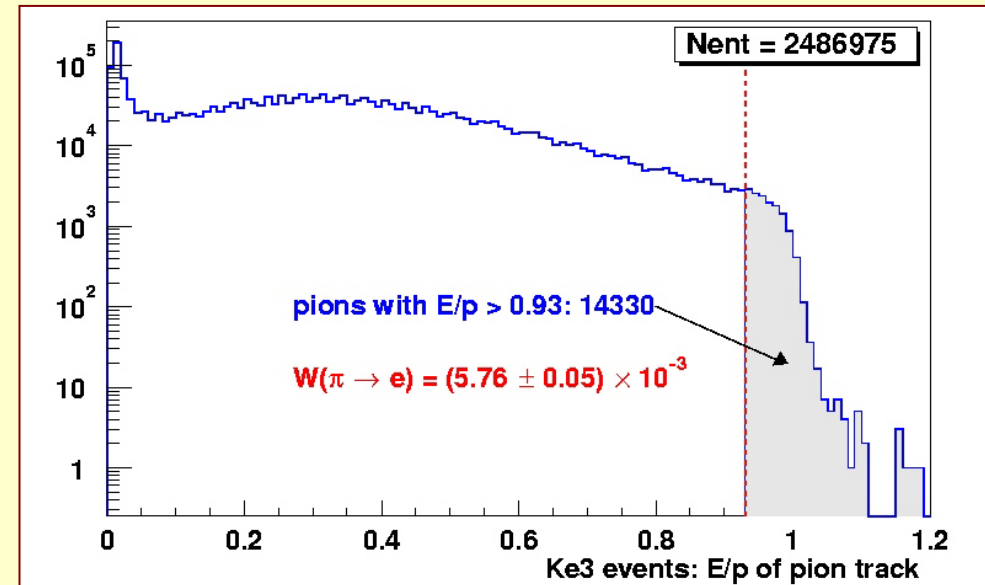
$$P(\pi \rightarrow e) = 5.8 \cdot 10^{-3}$$

## ❖ Inefficiency of electron ID

- Estimate  $e^\pm$  inefficiency from  $K_{e3}$  data with identified  $\pi^\pm$  ( $0.3 < E/p < 0.7$ )

$$P(e \rightarrow \pi) = 4.9 \cdot 10^{-3}$$

Selected 6.7 million  $K_{e3}$



## $K_L^0 \rightarrow \pi e \nu$

### ❖ Monte Carlo simulation of detector acceptance

- All two track channels involved – ( $Ke3$ ,  $K\mu3$ ,  $K3\pi$ ,  $K2\pi$ ,  $K3\pi^0_D$ )
- For average 2-track acceptance use Br fractions
- Average from PDG and KTeV ( $B\mu3/Be3$ ,  $B3\pi/Be3$ , ...)

$$A_{2tr} = 0.2412 \pm 0.0004$$

### ❖ Ke3 simulation includes radiative corrections and $Ke3\gamma$ with real photons Ginsberg (Phys.Rev. 171, 1675(1968)+ errata)

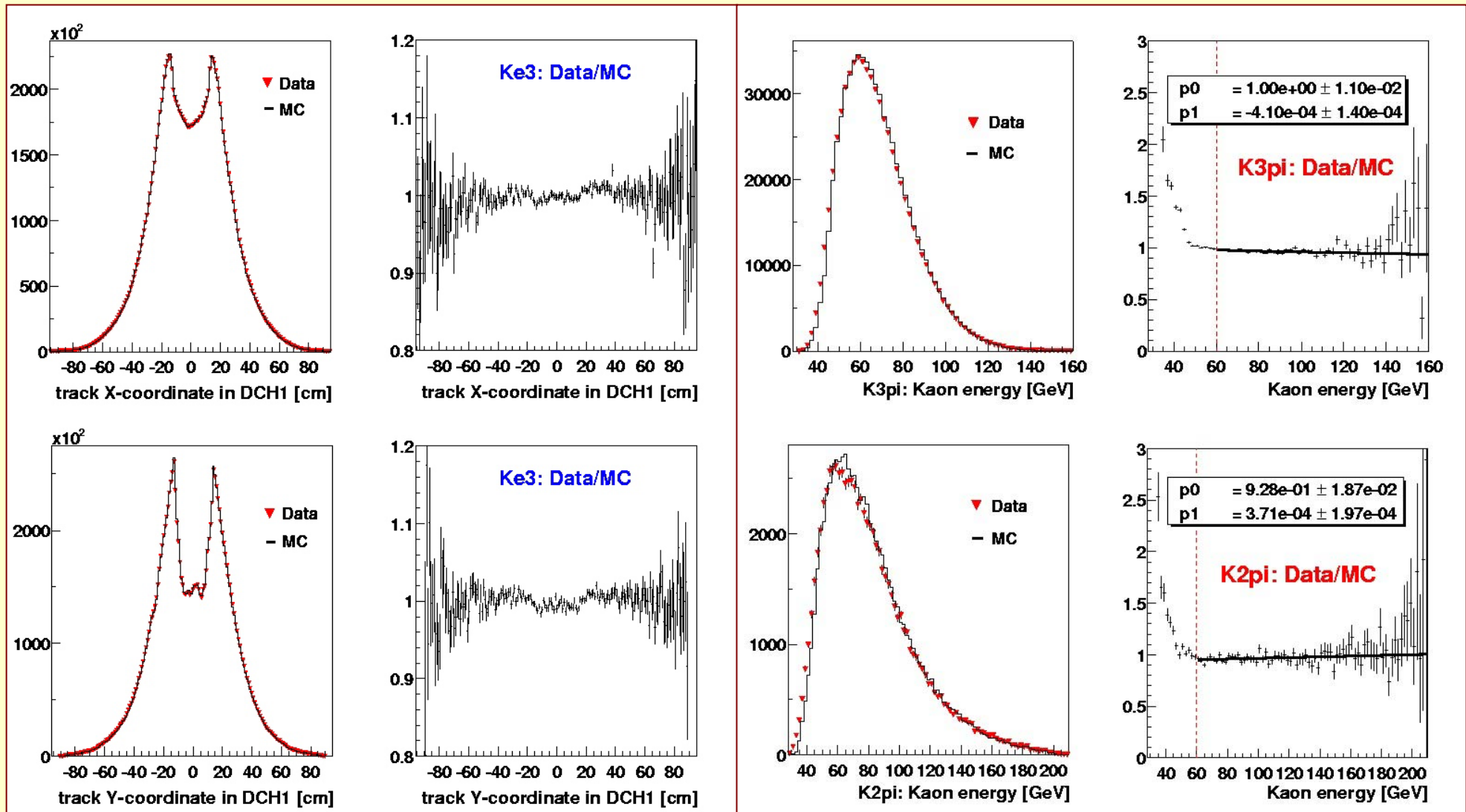
### ❖ Good agreement between MC and data except for high momentum $K_L^0$

### ❖ Systematic errors

- Main contribution comes from inexact knowledge of beam momentum (can be reconstructed only up to quadratic ambiguity)
- For measurement of beam momentum distribution –  $K2\pi$  and  $K3\pi$  decays
- Experimental uncertainty of 0.7% on measured ratio

### ❖ Statistical errors are negligible

# $K_L^0 \rightarrow \pi e \nu$





$$K_L^0 \rightarrow \pi e \nu$$

## Experimental result

$$\text{Br}(K_L^0 e3)/\text{Br}(2\pi) = 0.4978 \pm 0.0035$$

To determine  $\text{Br}(K_L^0 \rightarrow \pi e \nu)$  we need  $\text{Br}(K_L^0 \rightarrow 3\pi^0)$

PDG04:  $\text{Br}(K_L^0 \rightarrow 3\pi^0) = 0.2105 \pm 0.0028$

KTeV  $\text{Br}(K_L^0 \rightarrow 3\pi^0) = 0.1945 \pm 0.0018$  ?

Average according PDG prescription

$$\text{Br}(K_L^0 \rightarrow 3\pi^0) = 0.1992 \pm 0.0070$$

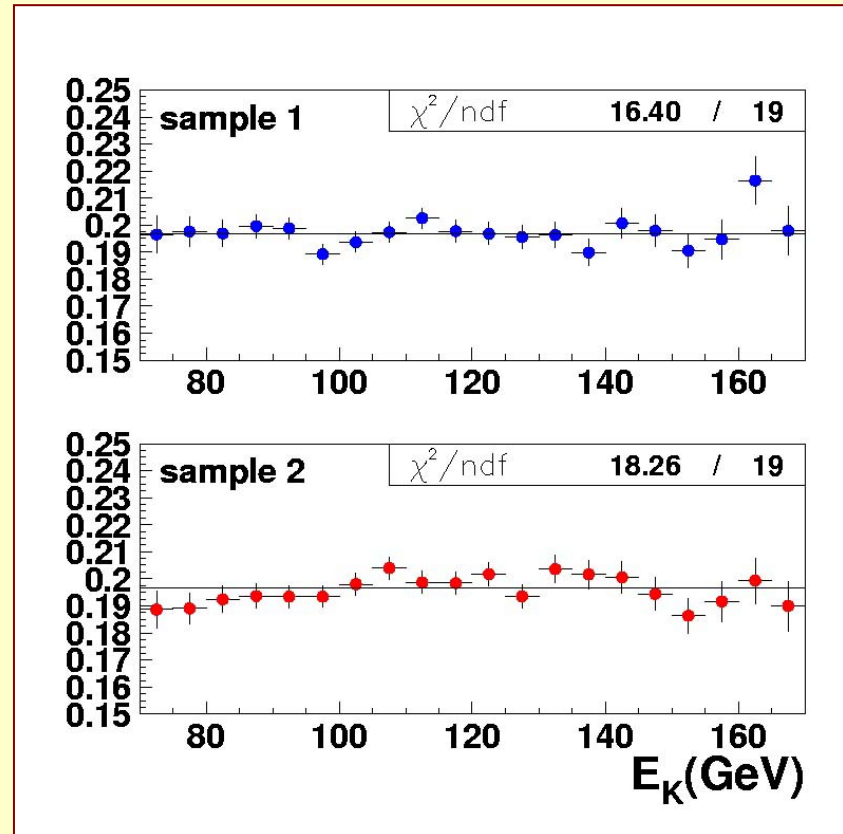
$$\text{Br}(K_L^0 e3) = 0.4010 \pm 0.0028_{\text{exp}} \pm 0.0035_{\text{norm}}$$

## Measurement of $\text{Br}(K_L^0 \rightarrow 3\pi^0)$

- ❖  $\text{Br}(K_L^0 \rightarrow 3\pi^0)$  is the main experimental uncertainty on  $\text{Br}(K_L^0 \rightarrow e^+e^-3\pi^0)$ 
  - PDG (Kreutz et al 1995) inconsistent with new KTeV result by  $\approx 5 \sigma$
  - Measure  $\text{Br}(K_L \rightarrow \pi^0\pi^0\pi^0) / \text{Br}(K_S \rightarrow \pi^0\pi^0)$
  - $\text{Br}(K_S \rightarrow \pi^0\pi^0) = 0.3104 \pm 0.0014$  well measured
- ❖ NA48/1 data, 2000:
  - High intensity  $K_S$  beam
  - No material (DCH etc) between collimator and LKr calorimeter
  - Ideal for measurement of neutral Kaon decays
- ❖ We used only small amount of 2000 data
  - $\sim 200\,000 K_L \rightarrow \pi^0\pi^0\pi^0$
  - $\sim 600\,000 K_S \rightarrow \pi^0\pi^0$
  - Two independent samples
  - Same number of  $K_L$  and  $K_S$  is produced on the target

## Measurement of $\text{Br}(K_L^0 \rightarrow 3\pi^0)$

- ❖ Main systematic
  - LKr energy scale  $\pm 0.0020$
  - Effective target position  $\pm 0.0017$
  - $K_L$  life time:  $\pm 0.0015$



$$\text{Br}(K_L \rightarrow \pi^0\pi^0\pi^0) = 0.1966 \pm 0.0006_{\text{stat}} \pm 0.0033_{\text{syst}}$$

Preliminary

In a good agreement with KTeV and KLOE results

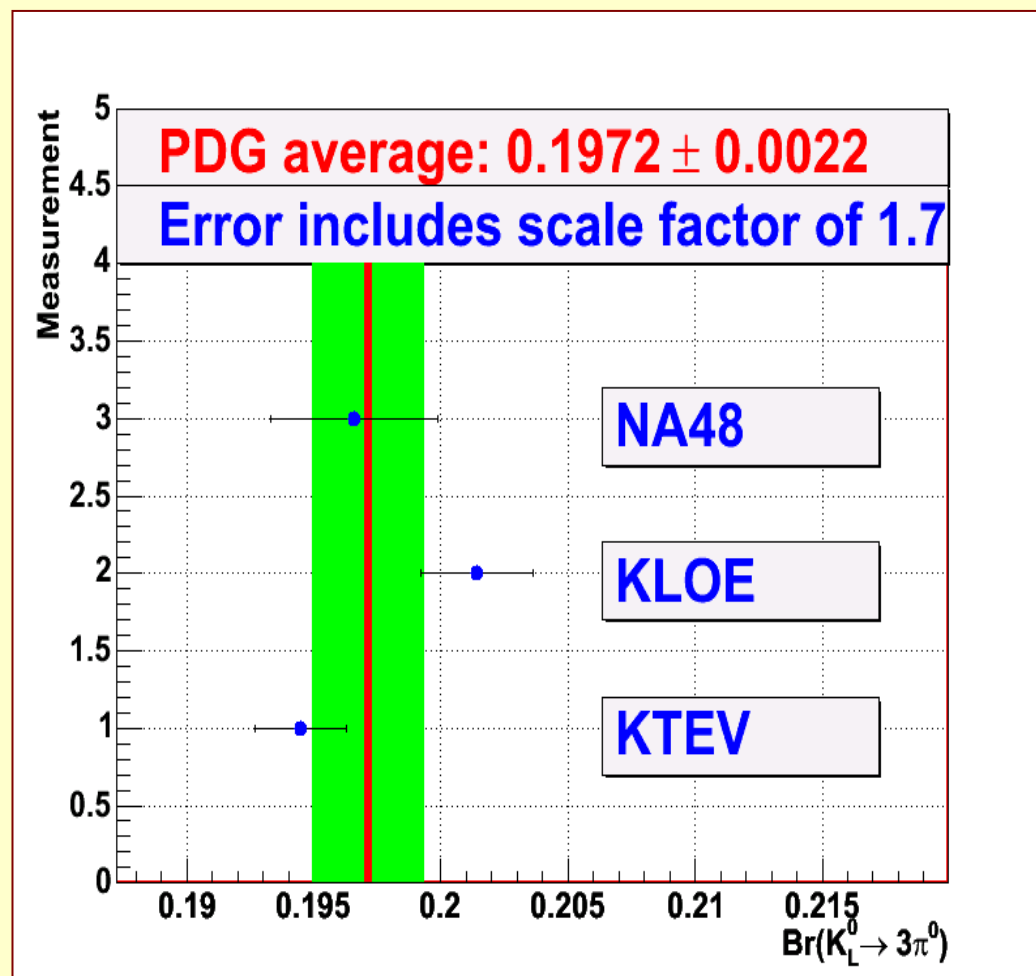
## Results for $\text{Br}(K_L^0 \rightarrow 3\pi^0)$

Taking into account the KTeV and KLOE results we obtain

$$\text{Br}(K_L \rightarrow \pi^0\pi^0\pi^0) = 0.1972 \pm 0.0022$$

Then the NA48 result changes to

$$\text{Br}(K_L^0 \rightarrow e^+e^-) = 0.4020 \pm 0.0030$$



## Measurement of $\text{Br}(\text{K}^\pm \rightarrow \pi^0 e^\pm \nu)$

- ❖ NA48/2 data from 2003

- Low intensity  $\text{K}^+/\text{K}^-$  run (8 hours) with minimum bias trigger

- ❖ Normalize  $\text{K}^\pm \rightarrow \pi^0 e^\pm \nu$  decay to  $\text{K}^\pm \rightarrow \pi^\pm \pi^0$

$$\text{Br}(\text{K}^\pm \rightarrow \pi^\pm \pi^0) = 0.2113 \pm 0.0014$$

- ❖ Selected events

$$\text{K}^+ \rightarrow \pi^0 e^+ \nu \quad 59\,000 \text{ ev.}$$

$$\text{K}^- \rightarrow \pi^0 e^- \nu \quad 33\,000 \text{ ev.}$$

$$\text{K}^+ \rightarrow \pi^+ \pi^0 \quad 468\,000 \text{ ev.}$$

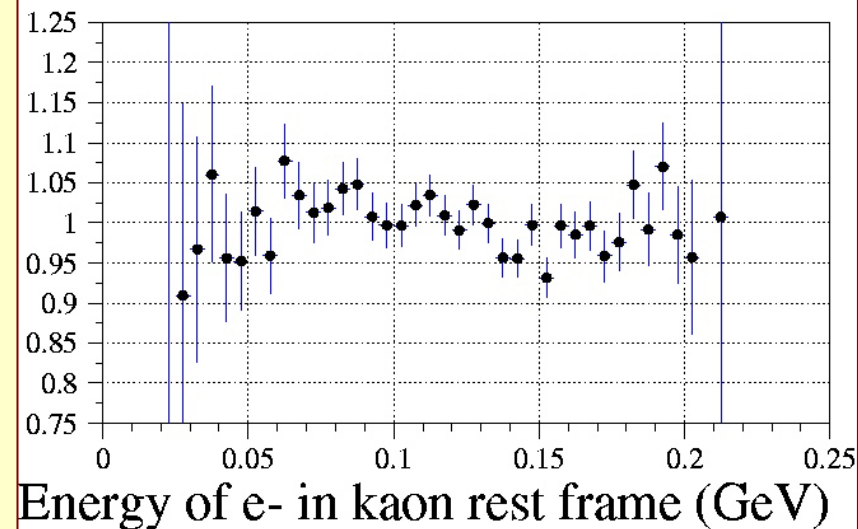
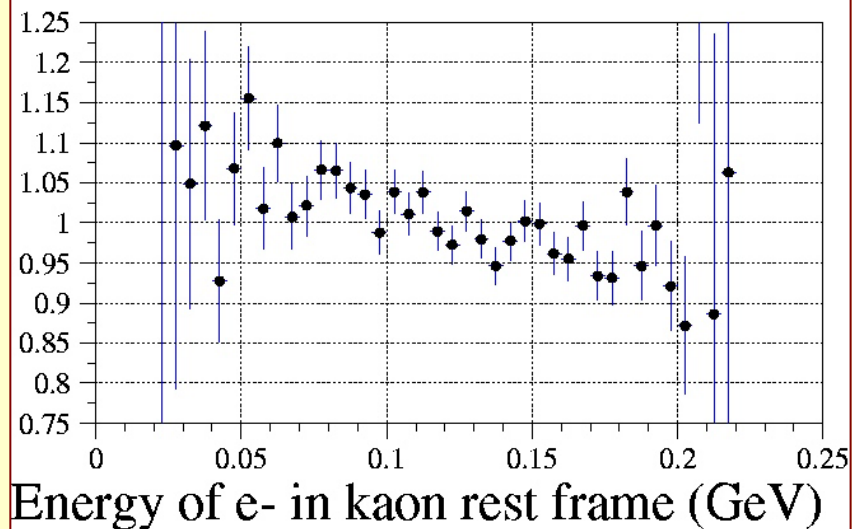
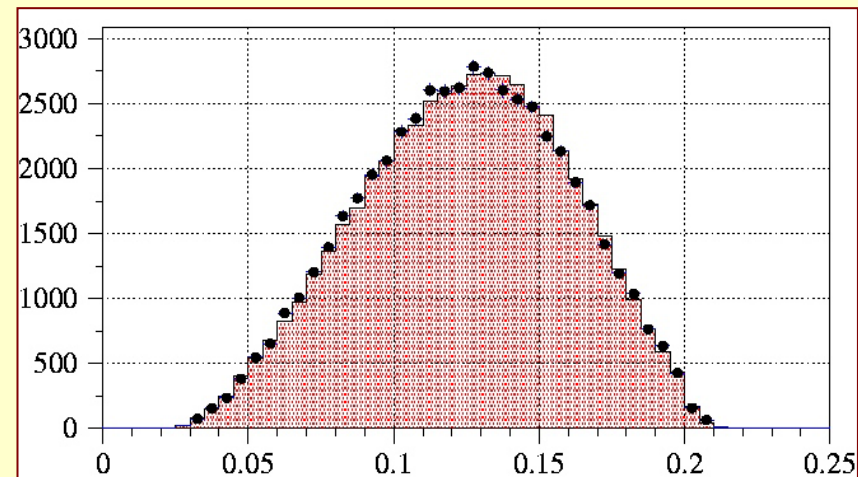
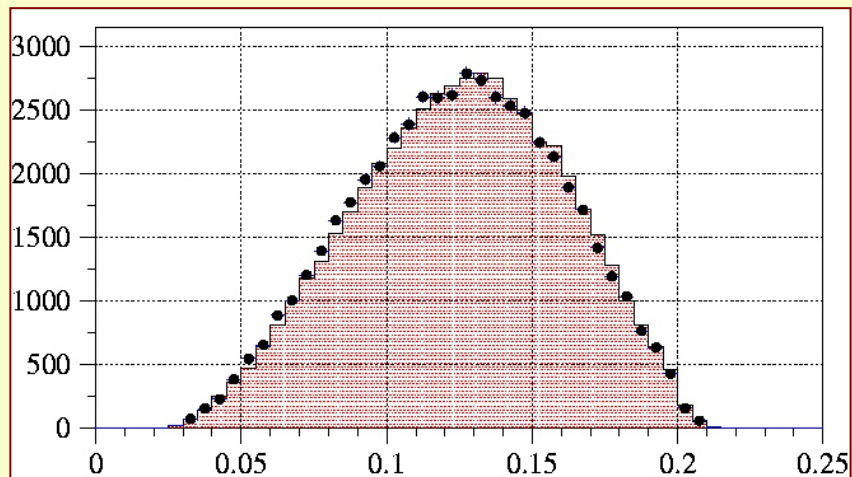
$$\text{K}^- \rightarrow \pi^- \pi^0 \quad 260\,000 \text{ ev.}$$

- ❖ Practically background free

- ❖ Systematic

- Main sources – Detector acceptance,  $\text{Br}(\text{K}^\pm \rightarrow \pi^\pm \pi^0)$ , MC statistic

# Measurement of $\text{Br}(\text{K}^\pm \rightarrow \pi^\pm e\nu)$



Without radiative corrections

With radiative corrections

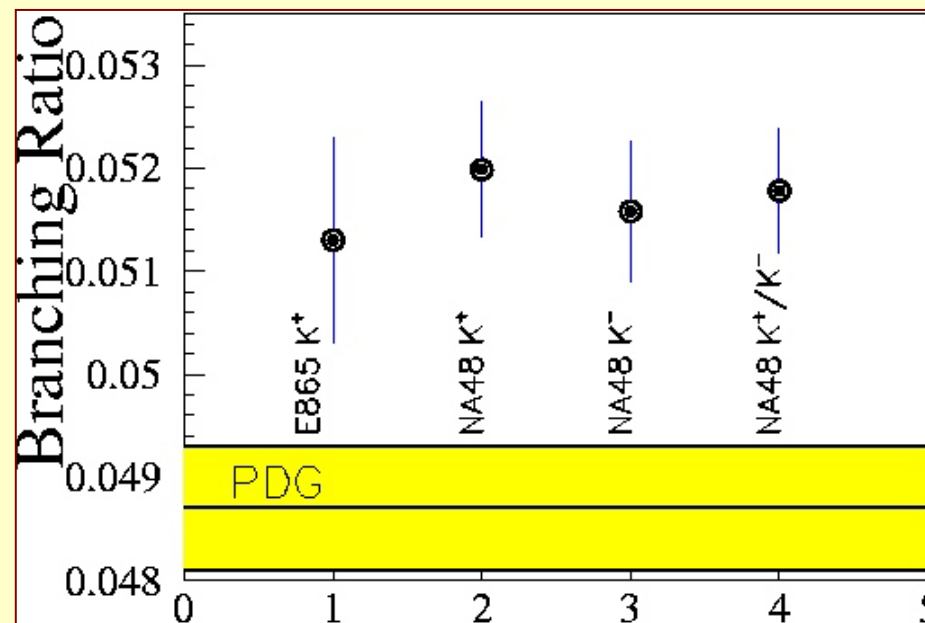
## Measurement of $\text{Br}(\text{K}^\pm \rightarrow \pi^0 e^\pm \nu)$

Preliminary NA48/2 result on  $\text{Br}(\text{K}^\pm \rightarrow \pi^0 e^\pm \nu)$

$$\text{Br}(\text{K}^+ \rightarrow \pi^0 e^+ \nu) = (5.163 \pm 0.021_{\text{stat}} \pm 0.056_{\text{syst}}) \%$$

$$\text{Br}(\text{K}^- \rightarrow \pi^0 e^- \nu) = (5.093 \pm 0.028_{\text{stat}} \pm 0.056_{\text{syst}}) \%$$

$$\text{Br}(\text{K}^\pm \rightarrow \pi^0 e^\pm \nu) = (5.14 \pm 0.02_{\text{stat}} \pm 0.06_{\text{syst}}) \%$$



## New NA48 results

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Other results



# Ke3 form factors

❖  $K_L \rightarrow \pi e \nu$  form factors - the same 1999 data

- 5.6 million reconstructed events
- Result for pure vector interaction

$$\lambda_+ = 0.0288 \pm 0.0005_{\text{stat}} \pm 0.0011_{\text{syst}}$$

- Result for 3-parameter fit:

$$\lambda_+ = 0.0284 \pm 0.0007 \pm 0.0013$$

$$\left| \frac{f_S}{f_+(0)} \right| = 0.015^{+0.007}_{-0.010} \pm 0.0012$$

$$\left| \frac{f_T}{f_+(0)} \right| = 0.05^{+0.03}_{-0.04} \pm 0.03$$

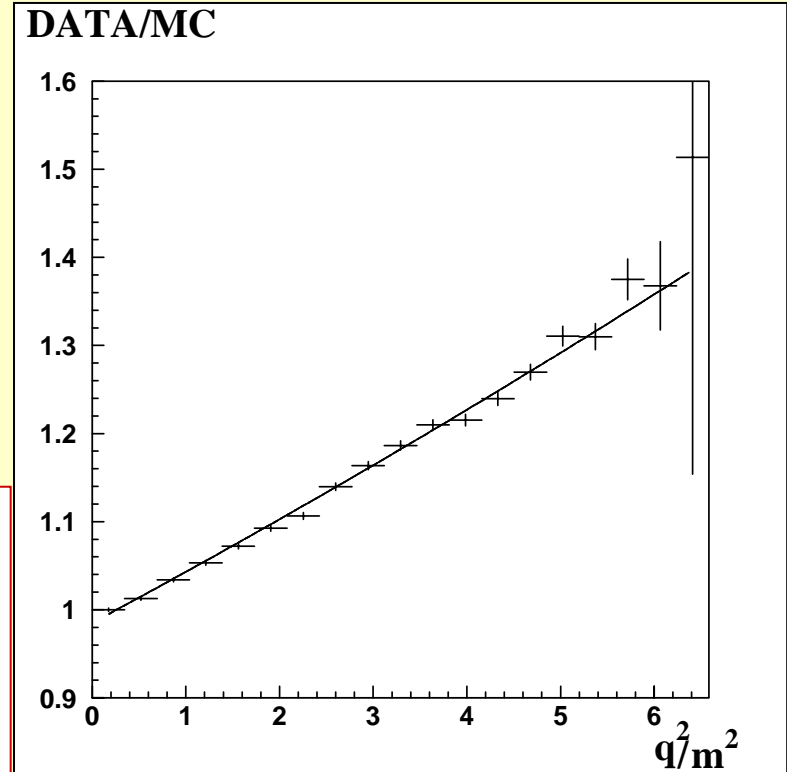
PDG(2004):

$$\lambda_+ = 0.0291 \pm 0.0018$$

$$\left| \frac{f_S}{f_+(0)} \right| < 0.04$$

$$\left| \frac{f_T}{f_+(0)} \right| < 0.23$$

Precise test of CKM unitarity



❖ No evidence for scalar and tensor couplings!

❖ No evidence for statistically significant quadratic term!

# $K_L^0 \rightarrow \pi e \nu \gamma$

## ❖ Data analysis

➤ Tight selection in order to suppress background from

$K_{3\pi}$ ,  $K_{e4}$  and

$K_{e3}$  + accidental photon

➤ Selected events:

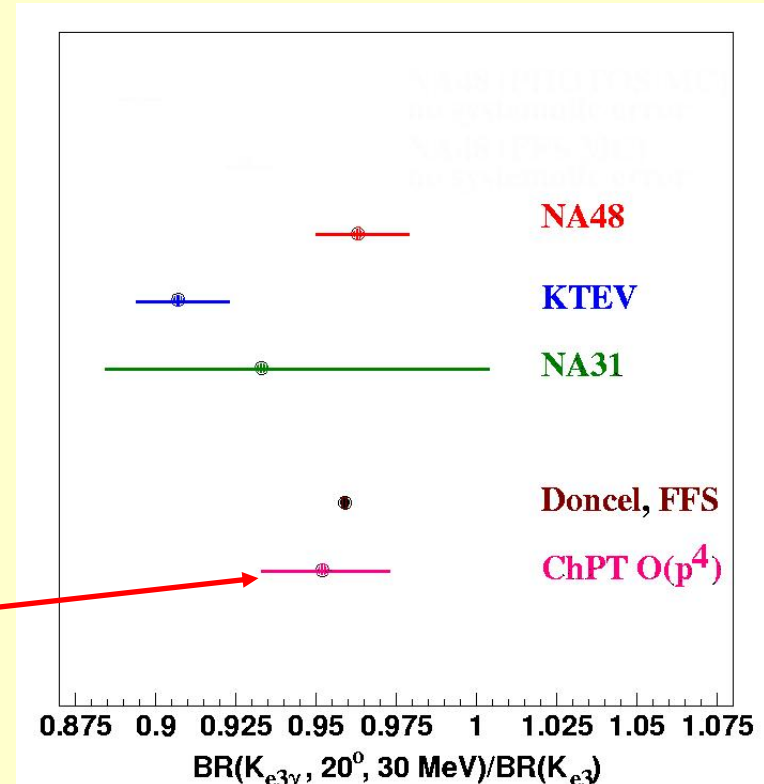
19000 :  $K_{e3\gamma}$

5.6 million :  $K_{e3}$

**Gasser et al.**

**Result:**

$$\frac{\Gamma(K_{e3\gamma}, E_\gamma^* > 30 \text{ MeV}, \theta_{e\gamma}^* > 20^\circ)}{\Gamma(K_{e3})} = (0.964 \pm 0.008^{+0.012}_{-0.011})\%$$



❖ **Good agreement with theory predictions!**

Precise test of CKM unitarity

## New NA48 results

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Determination of  $V_{us}$

## Determination of $V_{us}$

The physical quantity

$$\Gamma(K_{e3(\gamma)}) = \Gamma(K_{e3}) + \Gamma(K_{e3\gamma}) + \dots$$

where the radiative corrections with virtual and real photons are taken into account **Is well defined, calculable and measurable!**

The decay density distribution is given by

$$\rho(y, z) = NS_{EW} A(y, z) |f_+(t)|^2$$

where  $S_{EW}(M_\rho, M_Z)$  is a short-distance enhancement factor and

$$y=2E_\pi/M_K, \quad z=2E_e/M_K,$$

$$N = C^2 \frac{G_F^2 |V_{us}|^2 M_K^5}{128\pi^3}$$

## Determination of $V_{us}$

Then

$$\Gamma(K_{e3}(\gamma)) = NS_{EW} |f_+^{K\pi}(0)|^2 I_K$$

where  $I_K$  is the phase-space integral

$$I_K = \int_D dydz A(y, z) \frac{|f_+(t)|^2}{|f_+(0)|^2}$$

For linear parameterization of  $f_+(t)$  we have

$$I_K = a_0 + a_1 \lambda_+ + a_2 (\lambda_+)^2$$

and for quadratic parameterization of  $f_+(t)$   $I_K$  is given by

$$I_K = a_0 + a_1 \lambda_+ + a_2 (2\lambda'_+ + \lambda_+^2) + a_3 \lambda_+ \lambda'_+ + a_4 (\lambda'_+)^2$$

## Determination of $V_{us}$

$|V_{us}|$  can be extracted from  $K \rightarrow \pi e \nu$  via

$$|V_{us}| \cdot f_+^{K\pi}(0) = \sqrt{\frac{128 \pi^3 \Gamma(Ke3(\gamma))}{C^2 G_F^2 M_K^5 S_{EW} I_K}}$$

Where:

$S_{EW} = 1.0232$  – short distance enhancement factor,

$I_K(f_+^{K\pi}(t))$  – phase space integral,  $C = \begin{cases} 1 & K_{e3}^0 \\ 1/\sqrt{2} & K_{e3}^+ \end{cases}$

We followed the prescription for  $V_{us}$  determination proposed in V.Cirigliano, M. Knecht, H. Neufeld, H. Rupertsberger, P. Talavera, In Eur.Phys.J. C23 p121, 2002

V.Cirigliano, H. Neufeld, H. Pichl, Eur.Phys.J. C35 p53, 2004

**Important** – to treat all experimental data in the same way!

**Radiative corrections** (including virtual and real photons)! **A few %**

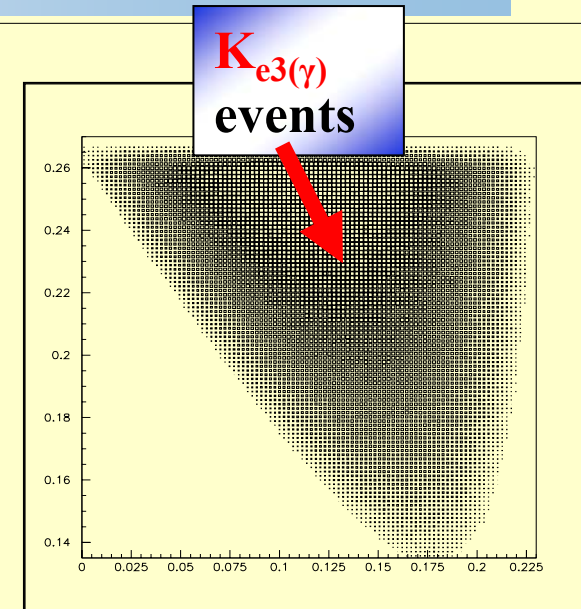
# Determination of $V_{us}$

## Prescription

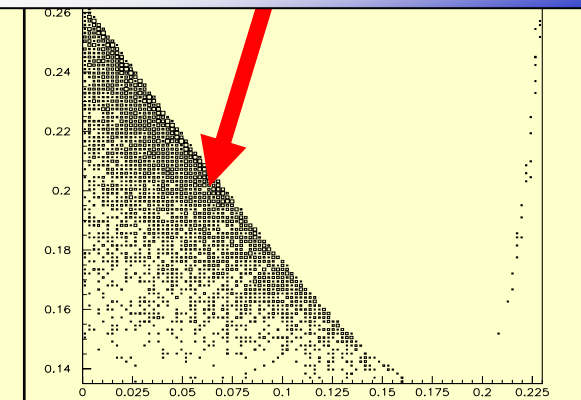
- Accept all photon energies
- Accept all angles between pion and positron
- Accept only pion and positron energies within the original 3-body Dalitz plot.
- Inclusive rate obtained by integrating over the original domain

Experimentally – inclusive measurement of  $\text{Br}(\text{Ke}3)$ . For determination of  $V_{us}$ , the corresponding  $\text{Br}(\text{Ke}3)$  should be inside the Dalitz plot. Corrections are:

$$C_K = 0.58\% \text{ for } K^0 \quad \text{and} \quad C_K = 0.56\% \text{ for } K^+$$



$K_{e3\gamma}$  events (excluded for the  $V_{us}$  extraction)



## Determination of $V_{us}$

### Input for calculation of $V_{us}$

#### Experimental data

- ✓ Br(Ke3)
- ✓ Mean life times of  $K_L^0$ ,  $K_S^0$ ,  $K^+$
- ✓ Linear and quadratic slopes of  $f_+(t) - \lambda_+$  and  $\lambda_+, \lambda_+'$
- Theoretical input –  $f_+^{K\pi}(0)$



## Determination of $V_{us}$

### Experimental data

To have comparable results  
Experimental data should be treated in the same way

- Inclusive measurement of the  $\text{Br}(\text{Ke3})$
- Correct account for radiative corrections, including real photons

**Two classes of data on measurement of  $\text{Br}(\text{Ke3})$**   
**Old data** – actually what is included in PDG 2004  
**New data** – published or reported in 2003 and 2004

## Determination of $V_{us}$

### Old experimental data - $K^\pm$

#### Direct measurement of $Br(K_{e3})$

- dominating experiment Chiang et al., Phys.Rev.D6, 1972, p.1254 accuracy  $\sim 2\%$   
Ke3 measurement is not inclusive  
No radiative corrections  
The decays  $\pi \rightarrow \mu$  are not taken into account  
The Dalitz decays of  $\pi^0$  are not taken into account

#### In the PDG fit also contribute

- $Br(K_{e3})/Br(2\pi)$   
-in the dominating experiment ( $\sim 5\%$  acc.) rad. corrections without real  $\gamma$
- $Br(K_{\mu 3})/Br(K_{e3})$   
K. Horie, Phys. Lett. B513, p. 311, 2001  
The measurement is not inclusive  
Ke3 $\gamma$  is considered as background

## Determination of $V_{us}$

### Old experimental data – $K_L^0$

**There is no direct measurement of  $\text{Br}(K_{e3})$   
In the PDG fit contribute**

- $\text{Br}(K_{\mu 3})/\text{Br}(K_{e3})$  – 4 experiments with good statistic
  - Two of them are perfect, both measure  $\text{Br}(K_{\mu 3})/\text{Br}(K_{e3}) = 0.662$  close to KTeV result
  - The other two – Hydrogen bubble chambers
    - In this case separation of  $K_{e3}$  and  $K_{\mu 3}$  decays is extremely difficult
    - 50% of the events are ambiguous – to separate complicated weighting procedure
    - Their results shift  $\text{Br}(K_{\mu 3})/\text{Br}(K_{e3})$  to 0.697
    - in strong disagreement with recent measurements
- The other contribution is from Kreutz, ZPHY C55, p.67, 1995 – the results from this experiment are in strong contradictions with recent measurements

## Determination of $V_{us}$

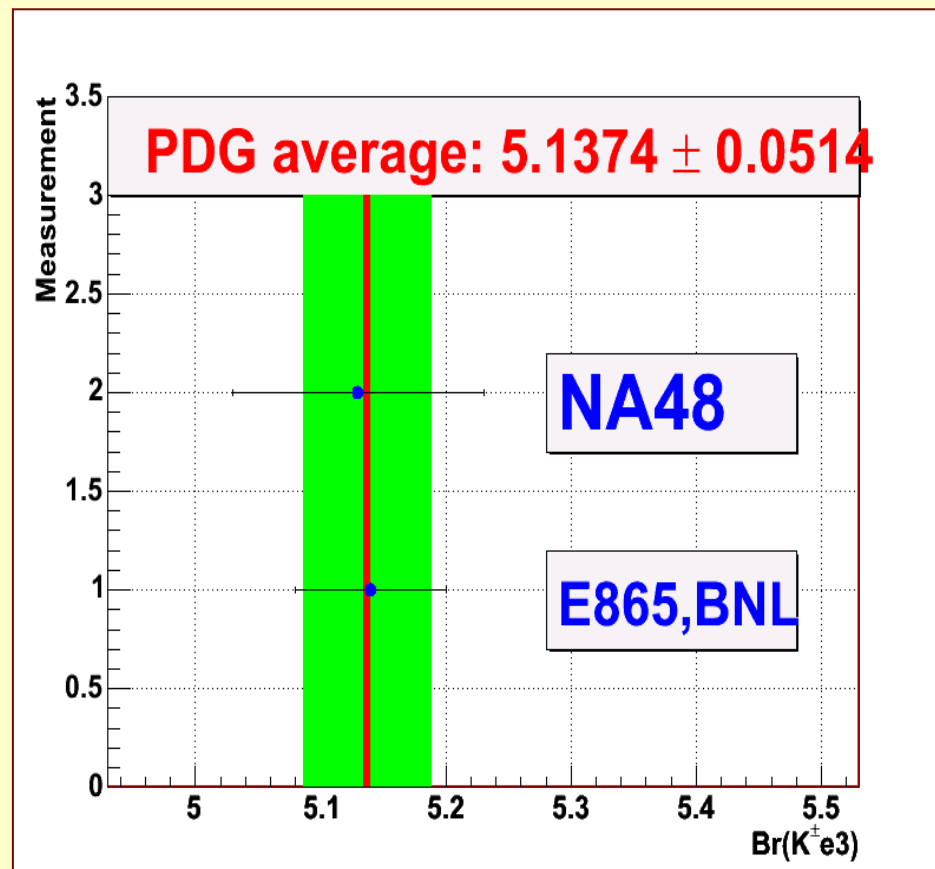
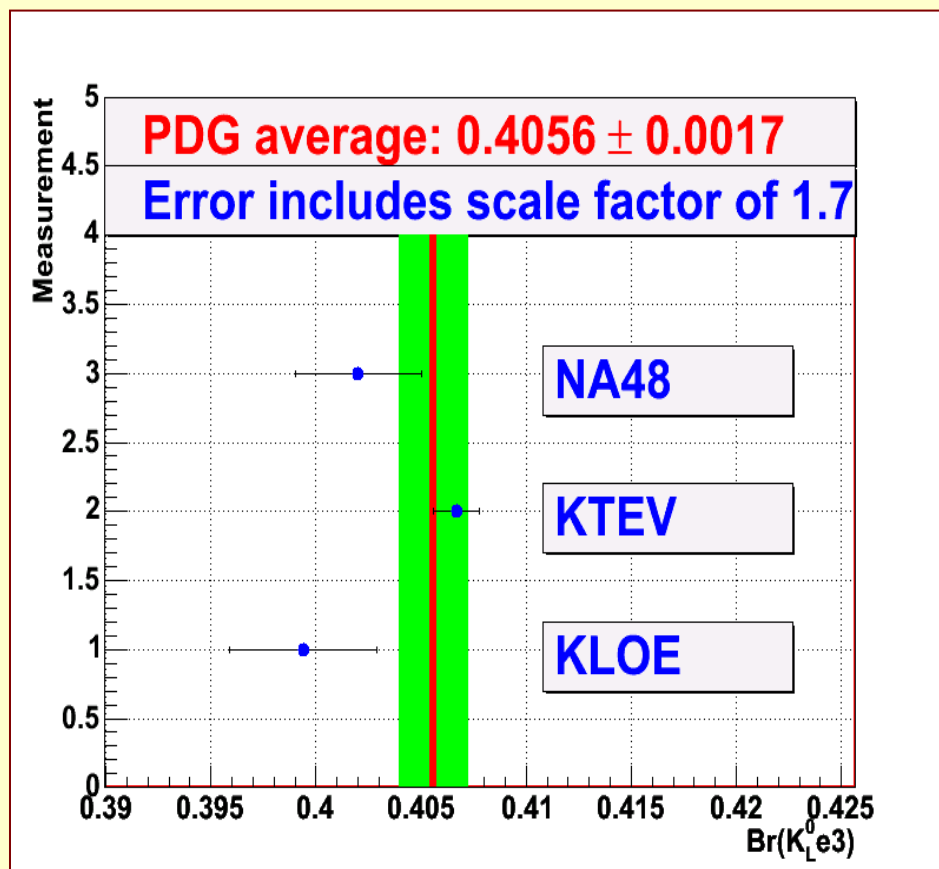
### Experimental data

The careful investigation of old experimental data on measurement of  $\text{Br}(\text{Ke}3)$  leads to the definite conclusion that due to different reasons they are not enough accurate and are not suitable for extraction of  $V_{us}$  matrix element

In what follows we will use only the new high statistics experimental data on measurement of  $\text{Br}(\text{Ke}3)$

# Determination of $V_{us}$

## Br(Ke3)



$$\text{Br}(K_L^0 e3) = 0.4056 \pm 0.0017$$

$$\text{Br}(K^\pm e3) = (5.137 \pm 0.051)\%$$

# Determination of $V_{us}$

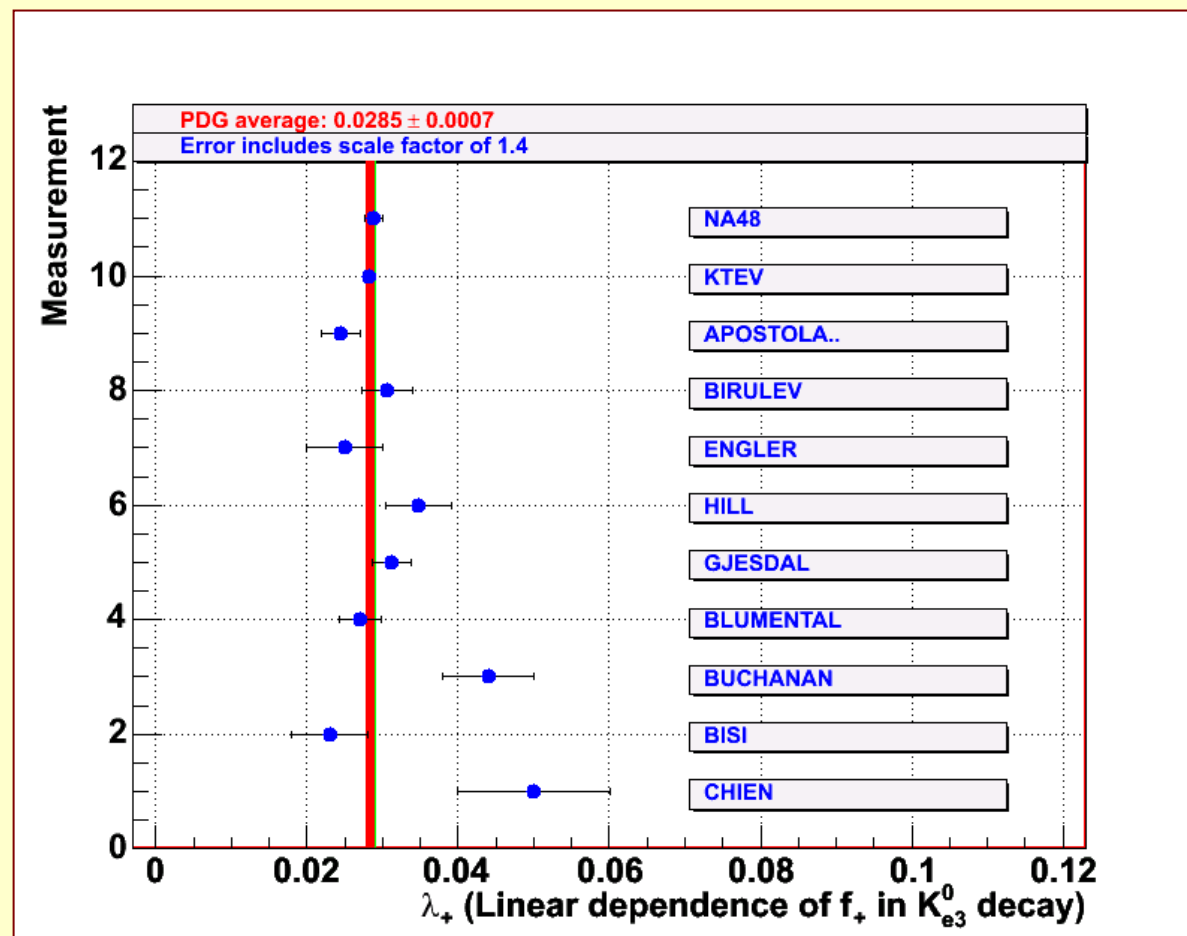
## Slope of the $f_+(t) - K_L^0$

Linear approximation

$$\lambda_+ = 0.0285 \pm 0.0007$$

Quadratic approximation  
KTeV result

$$\lambda_+ = 0.02167 \pm 0.00199$$
$$\lambda_+' = 0.00144 \pm 0.00039$$



# Determination of $V_{us}$

## Slope of the $f_+(t) - K^\pm$

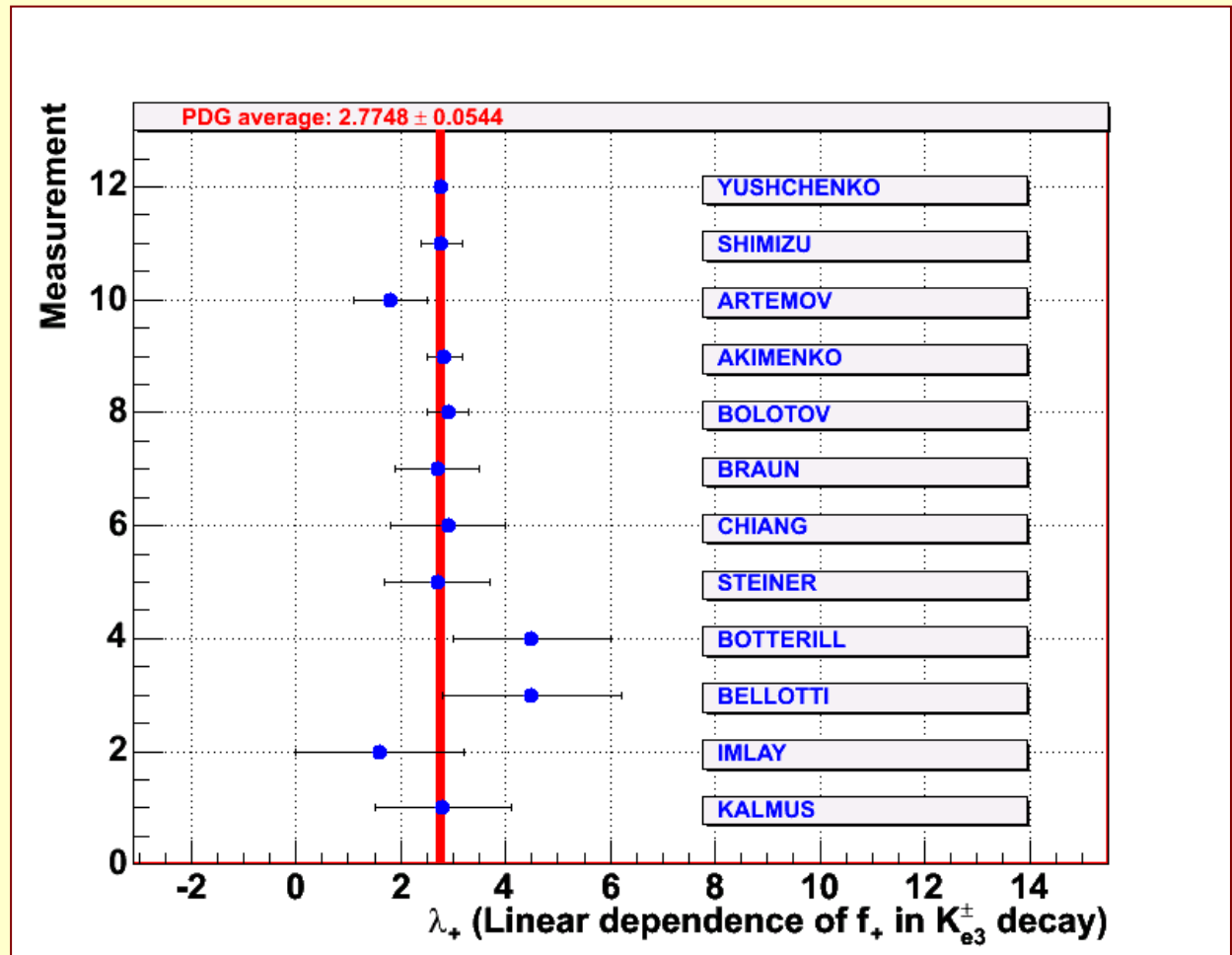
Linear approximation

$$\lambda_+ = 0.0277 \pm 0.0005$$

Quadratic approximation  
ISTRA+ result

$$\lambda_+ = 0.02324 \pm 0.00155$$

$$\lambda_+' = 0.00084 \pm 0.00041$$



# Determination of $V_{us}$

## Mean life time

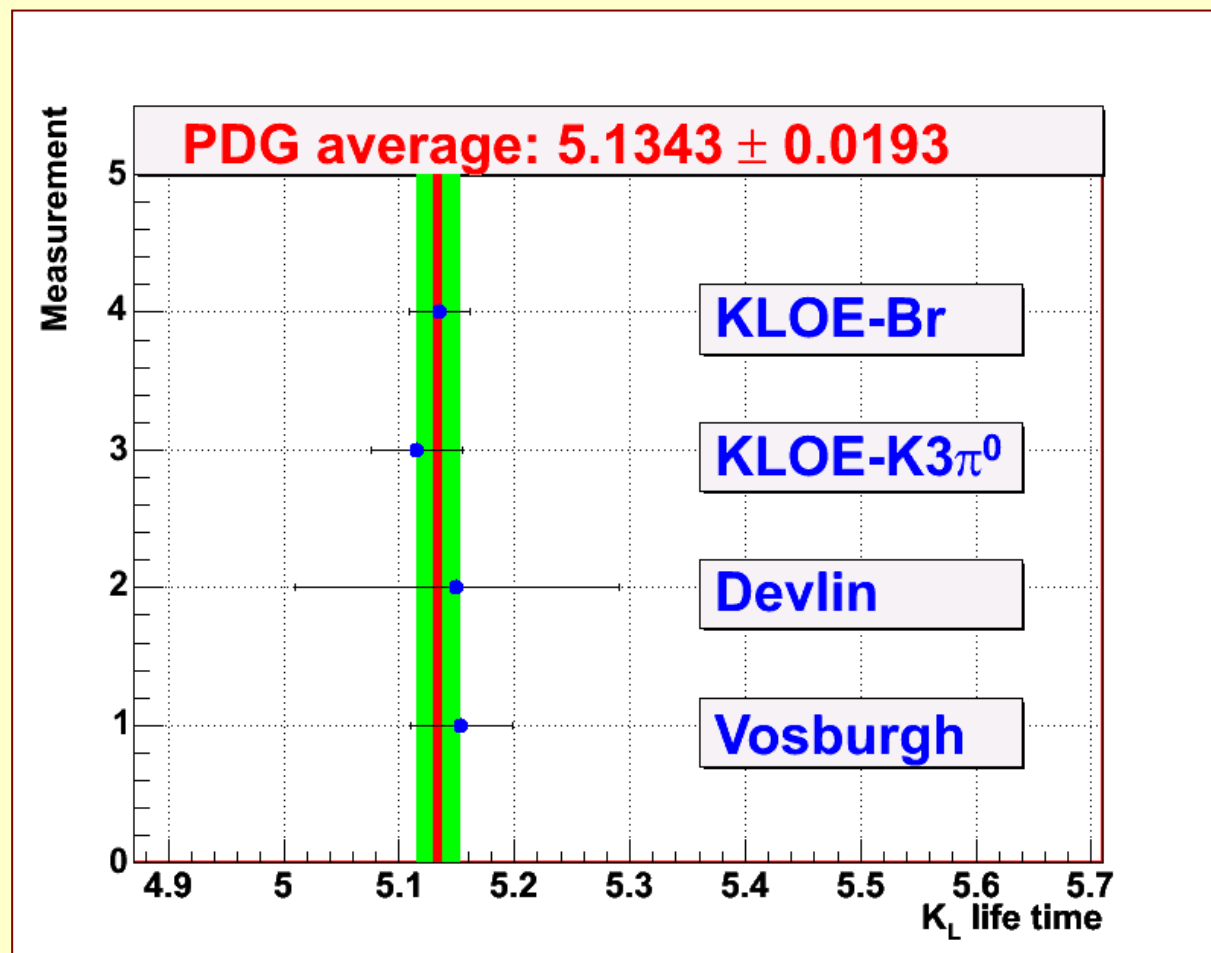
$K_L^0$

New KLOE results

$$\tau = (5.134 \pm 0.0193) \cdot 10^{-8} \text{s}$$

$K^\pm$

$$\tau = (1.2385 \pm 0.0025) \cdot 10^{-8} \text{s}$$





## Determination of $V_{us}$

Calculation of  $f_+^{K\pi}(0)$

Let us represent  $f_+(t)$  in the following form:

$$f_+ = \tilde{f}_+(t) + \hat{f}_+(t)$$

$$\tilde{f}_+(p^4)$$

$$\tilde{f}_+(p^6)$$

QCD effects to  $O(p^6)$   
EM contribution to  $O(e^2p^2)$   
EM contraterms relevant to  $\pi^0 - \eta$  mixing

local effects of virtual photon  
exchange of order  $O(e^2p^2)$

## Determination of $V_{us}$

### Calculation of $f_+^{K\pi}(0)$

Calculation of  $f_+(0)$  to  $O(p^4)$  - Gasser & Leutwyler  
First calculation to  $O(p^6)$  – Leutwyler & Roos  
QCD + isospin breaking

$$\tilde{f}_+^{K^0\pi^-}(0) = 0.961 \pm 0.008$$

$$\tilde{f}_+^{K^+\pi^0}(0) = 0.982 \pm 0.008$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -0.016 \pm 0.008$$

### Bijnens & Talavera

$$\tilde{f}_+^{K\pi}(0) = 0.976 \pm 0.010$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -8 \left( \frac{M_K^2 - M_\pi^2}{F_\pi^2} \right) [C_{12}^r(\mu) + C_{34}^r(\mu)] + \Delta_{loops}(\mu)$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6}^{local} = -0.016 \pm 0.008$$

$$\Delta_{loops}(M_\rho) = 0.0146 \pm 0.0064$$

## Determination of $V_{us}$

### Calculation of $f_+^{K\pi}(0)$

**Quenched lattice calculations – Becirevic et al.**

$$\tilde{f}_+^{K^0\pi^-}(0) = 0.960 \pm 0.009$$

$$\tilde{f}_+^{K\pi}(0)|_{p^6} = -0.017 \pm 0.008$$

**Cirigliano, Neufeld and Pichl**

**Calculation using  $\chi$ PT with virtual photons and leptons**

- **Isospin breaking by the quark masses up to  $O((m_u - m_d)p^2)$**
- **Isospin conserving contribution from SU(3) breaking  $O(p^6)$**
- **Electromagnetic effects up to  $O(e^2p^2)$**

$$f_+^{K^0\pi^+}(0) = 0.981 \pm 0.010$$

$$f_+^{K^+\pi^0}(0) = 1.002 \pm 0.010$$

## Determination of $V_{us}$

### Calculation of $f_+^{K\pi}(0)$

To extract  $V_{us}$  we have used the following values

	LO + NLO QCD	EM. radiative corrections	NNLO QCD	total
$K^0$	$0.97699 \pm 0.00002$	$0.0046 \pm 0.0008$	$-0.001 \pm 0.010$	$0.981 \pm 0.010$
$K^+$	$1.0002 \pm 0.0022$	$0.0032 \pm 0.0016$	$-0.001 \pm 0.010$	$1.002 \pm 0.010$
$K^0$	$0.97699 \pm 0.00002$	$0.0046 \pm 0.0008$	$-0.017 \pm 0.009$	$0.965 \pm 0.009$
$K^+$	$1.0002 \pm 0.0022$	$0.0032 \pm 0.0016$	$-0.017 \pm 0.009$	$0.986 \pm 0.010$

The main uncertainty ( $\sim 1\%$ ) comes from  $O(p^6)$  contribution

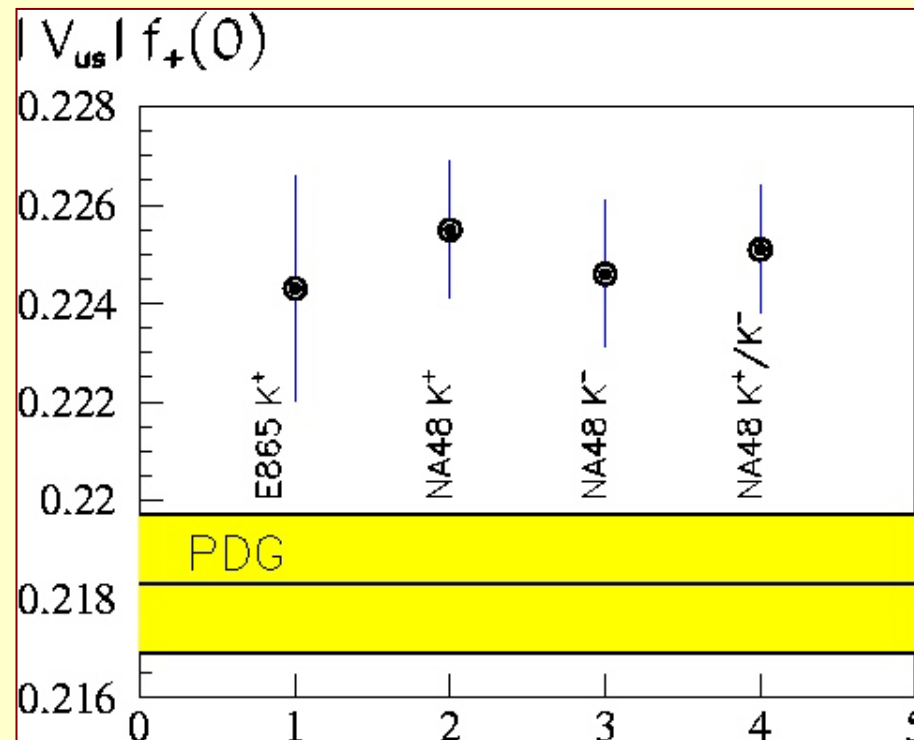
## Determination of $V_{us}f_+(0)$ – NA48

$$|V_{us}| \cdot f_+^{K^0\pi^+}(0) = 0.2146 \pm 0.0016$$

$$|V_{us}| \cdot f_+^{K^+\pi^0}(0) = 0.2250 \pm 0.0013$$

$$|V_{us}| \cdot f_+^{K^-\pi^0}(0) = 0.2235 \pm 0.0014$$

$$|V_{us}| \cdot f_+^{K^\pm\pi^0}(0) = 0.2245 \pm 0.0013$$



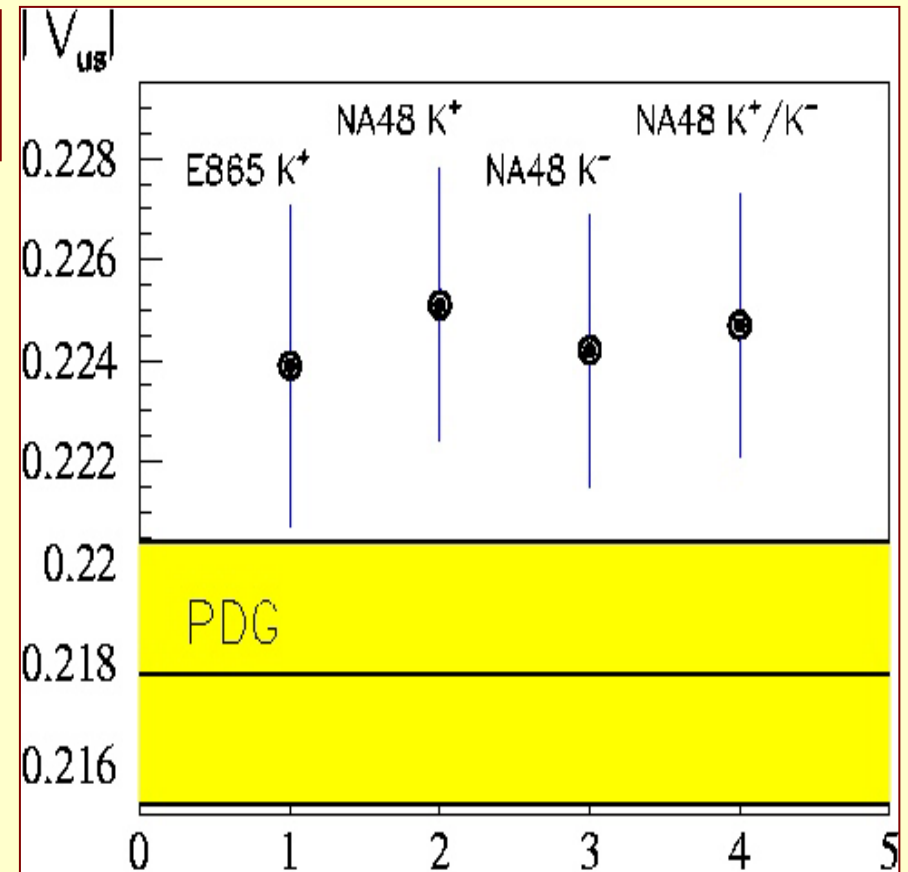
## Determination of $V_{us}$ - NA48

$$|V_{us}|^{K^0\pi^+}(0) = 0.2187 \pm 0.0016_{\text{exp}} \pm 0.0023_{\text{theor}}$$

$$|V_{us}|^{K^+\pi^0}(0) = 0.2246 \pm 0.0013_{\text{exp}} \pm 0.0023_{\text{theor}}$$

$$|V_{us}|^{K^-\pi^0}(0) = 0.2231 \pm 0.0014_{\text{exp}} \pm 0.0023_{\text{theor}}$$

$$|V_{us}|^{K^\pm\pi^0}(0) = 0.2241 \pm 0.0013_{\text{exp}} \pm 0.0023_{\text{theor}}$$



**Uncertainty in  $V_{us}$  is dominated by the theory!**

## Determination of $V_{us}$

### Results – $K^0$

#### Linear approximation of $f_+(0)$

$$f_+^{K^0\pi^+}(0) = 0.981 \pm 0.010$$

Experiment	Br	$V_{us} f_+(0)$	$V_{us}$
PDG	$0.3881 \pm 0.0027$	$0.2116 \pm 0.0009$	$0.2157 \pm 0.0024$
NA48	$0.4020 \pm 0.0030$	$0.2154 \pm 0.0009$	$0.2196 \pm 0.0024$
KTeV	$0.4067 \pm 0.0011$	$0.2166 \pm 0.0006$	$0.2208 \pm 0.0023$
KLOE – $K_L$	$0.3994 \pm 0.0035$	$0.2147 \pm 0.0011$	$0.2188 \pm 0.0025$
KLOE – $K_S$	$(7.09 \pm 0.11) \cdot 10^{-4}$	$0.2165 \pm 0.0017$	$0.2208 \pm 0.0022$
Average $K_L$	$0.4056 \pm 0.0017$	$0.2164 \pm 0.0007$	$0.2206 \pm 0.0024$

## Determination of $V_{us}$

### Results – $K^+$

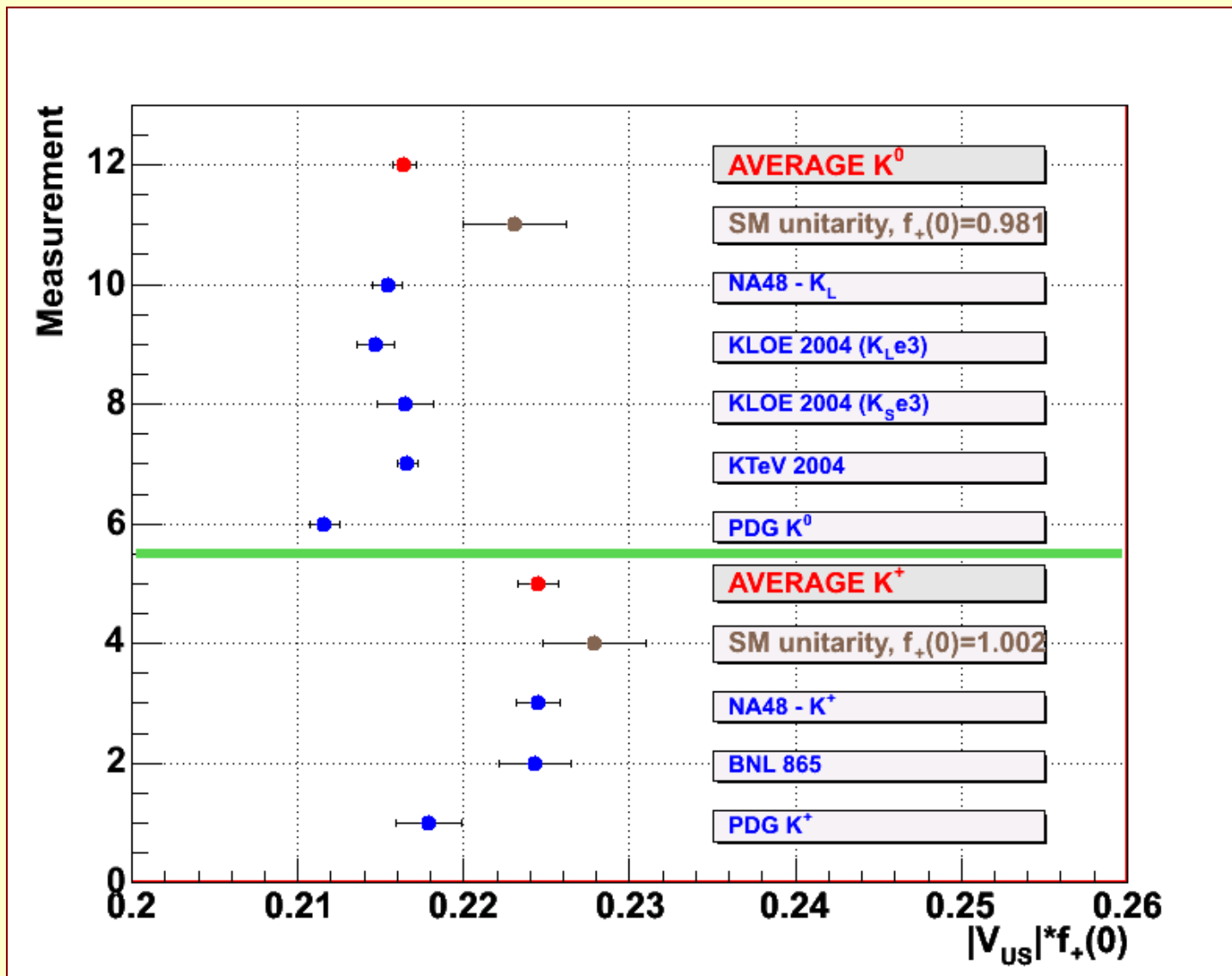
#### Linear approximation of $f_+(0)$

$$f_+^{K^+\pi^0}(0) = 1.002 \pm 0.010$$

Experiment	Br [%]	$V_{us}f_+(0)$	$V_{us}$
PDG	$4.84 \pm 0.09$	$0.2179 \pm 0.0020$	$0.2174 \pm 0.0030$
NA48	$5.14 \pm 0.06$	$0.2245 \pm 0.0013$	$0.2241 \pm 0.0026$
E865	$5.13 \pm 0.10$	$0.2243 \pm 0.0022$	$0.2239 \pm 0.0031$
Average	$5.137 \pm 0.051$	$0.2245 \pm 0.0012$	$0.2240 \pm 0.0025$



# Determination of $V_{us}$



## Consistency of $K_L$ and $K_{ch}$ data

Ratio of  $f_+(0)$  for  $K_L$  and  $K^+$  can be measured

$$R = f_+^{K^0\pi^+}(0) / f_+^{K^+\pi^0}(0)$$

Its calculation is free from many of the theoretical uncertainties

$$R^{th} = 1.022 \pm 0.003 - 16\pi\alpha X_1$$

$$1.017 \leq R^{th} \leq 1.027$$

From the averaged  $K_L$  and  $K^+$  data we obtain

$$R^{exp} = 1.038 \pm 0.006$$

- In disagreement with theoretical predictions  $\sim 2\sigma$**
- failure of the naïve dimensional analysis for  $X_1$
  - failure of chiral power counting
  - wrong mean life times

## Determination of $V_{us}$

**If we use for  $V_{us}$  determination**

$$f_+^{K^0\pi^+}(0) = 0.965 \pm 0.009$$

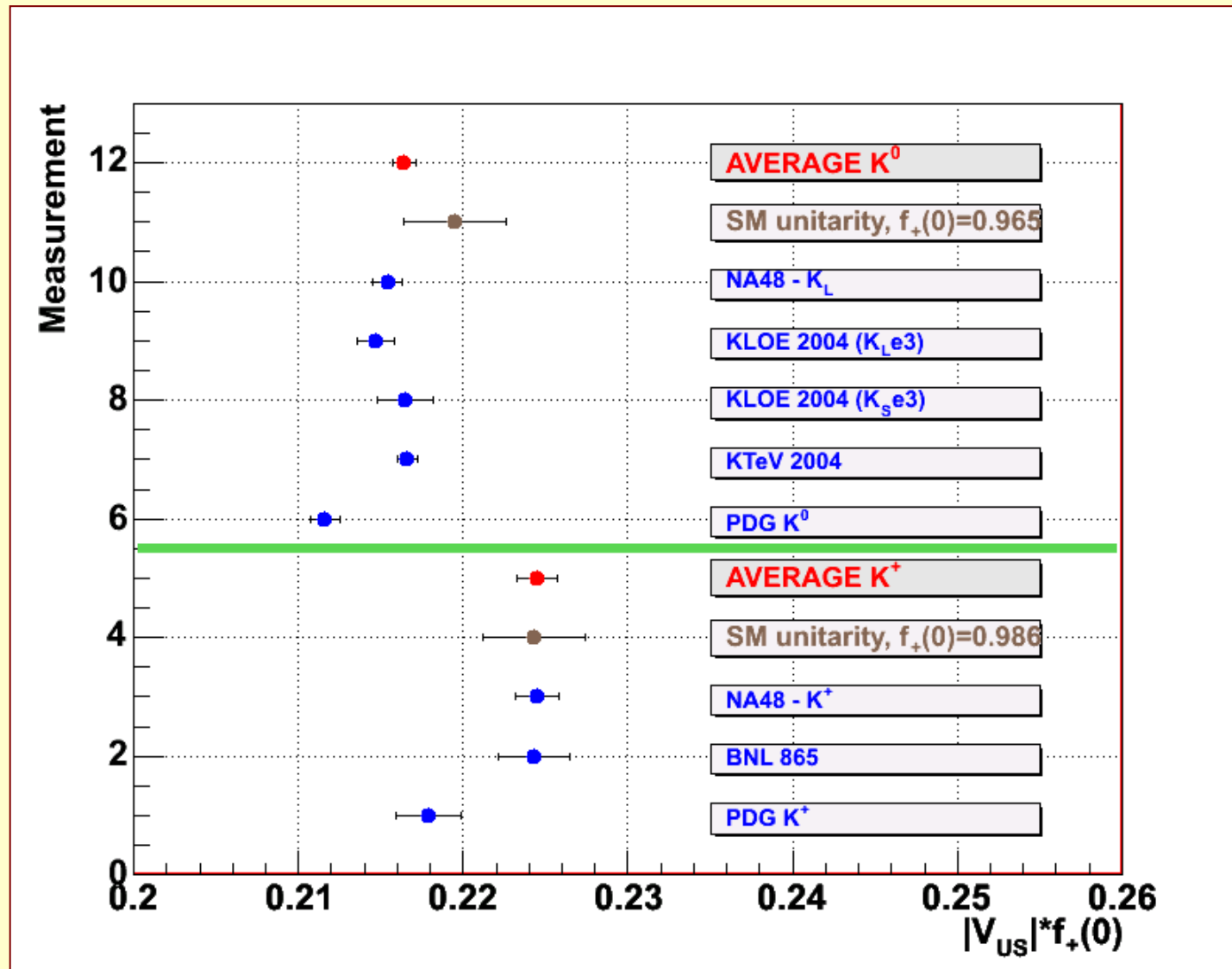
$$f_+^{K^+\pi^0}(0) = 0.986 \pm 0.010$$

$$|V_{us}|^{K^0\pi^+} = 0.2242 \pm 0.0022$$

$$|V_{us}|^{K^\pm\pi^0} = 0.2277 \pm 0.0026$$

**The values of  $V_{us}$  are changed  $\sim 2\sigma$**   
**Calculation of  $f_+(0)$  is the most important problem to be solved**

# Determination of $V_{us}$



## Determination of $V_{us}$

**Non linear approximation leads to**

$$|V_{us}| \cdot f_+^{K^0\pi^+}(0) = 0.2174 \pm 0.0010$$

$$|V_{us}| \cdot f_+^{K^\pm\pi^0}(0) = 0.2253 \pm 0.0014$$

$$|V_{us}|^{K^0\pi^+}(0) = 0.2216 \pm 0.0025$$

$$|V_{us}|^{K^\pm\pi^0}(0) = 0.2248 \pm 0.0026$$

**The values of  $V_{us}f_+(0)$  are changed  $\sim 1\sigma$  and  $\sim 0.6\sigma$**   
**The values of  $V_{us}$  are changed  $\sim 0.4\sigma$  and  $\sim 0.3\sigma$**

**and with the second set  $f_+(0)$**

$$|V_{us}|^{K^0\pi^+}(0) = 0.2253 \pm 0.0024$$

$$|V_{us}|^{K^\pm\pi^0}(0) = 0.2285 \pm 0.0027$$

**In perfect agreement with unitarity of CKM matrix**

## Determination of $f_+(0)$

If we suppose that CKM matrix is unitary

$$|V_{us}| = 0.2274 \pm 0.0021$$

then we can determine the values of  $f_+(0)$  using

$$|V_{us}| \cdot f_+^{K^0\pi^+}(0) = 0.2164 \pm 0.0007$$

$$|V_{us}| \cdot f_+^{K^+\pi^0}(0) = 0.2245 \pm 0.0012$$

$$f_+^{K^0\pi^+}(0) = 0.952 \pm 0.009$$

$$f_+^{K^+\pi^0}(0) = 0.987 \pm 0.010$$

The non linear approximation does not effects the result significantly

$$f_+^{K^0\pi^+}(0) = 0.9561 \pm 0.0086$$

$$f_+^{K^+\pi^0}(0) = 0.991 \pm 0.011$$

## Conclusions –NA48 results

- ❖ PDG values on  $|V_{us}|$  are in poor agreement with unitarity of the CKM matrix
- ❖ NA48 has performed  $|V_{us}|$  measurements in  $K^0_L e3$  and  $K^\pm e3$  decays
- ❖  $K_L$  and  $K^\pm$  results are
  - in disagreement with previous PDG values
  - in good agreement with recent results from KTeV, KLOE and E865
  - in fair agreement with SM predictions (better for  $K^\pm$ , worse for  $K_L$ )
  - different values from  $K^\pm$  and  $K_L$  ?!
- ❖ Semileptonic  $K_L$  decays
  - $K^0_L \rightarrow \pi e \nu$  form factors
  - Branching fraction for the radiative decay  $K^0_L \rightarrow \pi e \nu \gamma$
- ❖ More precise values for  $f_+(0)$  are needed to solve the unitarity dilemma

## Conclusions- $V_{us}$

- ❖ The careful analysis of the existing data has shown
  - the old measurements are not suitable for determination of  $V_{us}$
  - new measurement of all kaon branching fractions is desirable
  - new more precise measurements of  $Ke3$  and  $K\mu3$  form factors are needed
  - new measurement of the kaon mean life times will be welcome
  - KLOE and part of NA48 data are still preliminary
- ❖  $V_{us}$  values obtained using average values of  $Br(Ke3)$ 
  - Support the unitarity of CKM matrix
  - Strongly depend from the values of  $f_+(0)$
  - More precise calculation of  $O(p^6)$  contribution is required
- ❖ The experimental data for  $R$  are in disagreement with theoretical predictions
- ❖ Measured values of  $f_+(0)$  (with unitary CKM matrix) cause questions to the theory