Beyond the Standard Model

Lecture 5

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MSSM

SUSY SM

spin 0	spin 1/2	spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\tilde{u}_L, \tilde{d}_L$	u_L, d_L		3	2	$+\frac{1}{3}$
$\tilde{u}_R$	u_R		3	1	$+\frac{4}{3}$
$\tilde{d}_R$	d_R		3	1	$-\frac{2}{3}$
$\tilde{\nu}, \tilde{e}_L$	ν, e_L		1	2	-1
$\tilde{e}_R$	e_R		1	1	-2
H_u^+, H_u^0	$\tilde{h}_u^+, \tilde{h}_u^0$		1	2	$+1$
H_d^0, H_d^-	$\tilde{h}_d^0, \tilde{h}_d^-$		1	2	-1
	$\tilde{g}$	g	8	1	0
	$\tilde{w}^\pm, \tilde{w}^0$	$W^\pm, W^0$	1	3	0
	$\tilde{b}^0$	B^0	1	1	0

Building blocks

Matter fields (i.e. quarks, leptons and Higgs)
Chiral multiplets – ϕ, ψ, F

Recipe

for non-gauge interaction terms of a set of chiral supermultiplets ϕ_i, ψ_i, F_i:

1. Give a superpotential

$$W = \frac{1}{2} M_{ij}\, \phi_i \phi_j + \frac{1}{6} y_{ijk}\, \phi_i \phi_j \phi_k,$$

   with M_{ij} and y_{ijk} completely symmetric.

2. Build the interaction lagrangian as

$$\mathcal{L}_{\rm ng} = -\frac{1}{2} W_{ij}\, \psi_i \psi_j + W_i\, F_i + \text{h.c.}$$

   where

$$W_i = \frac{\partial W}{\partial \phi_i} \qquad W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

Gauge interactions
Vector multiplets – λ, A_μ, D

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} + g\sqrt{2}\left(\phi^* T^a \psi \lambda^a + \phi T^a \lambda^{a\dagger} \psi^\dagger\right) + g(\phi^* T^a \phi) D^a$$

is supersymmetric, provided the superpotential is gauge-invariant:

$$\delta_{\text{gauge}} W = W_i (T^a)_{ij} \phi_j = 0$$

The equations of motion for D^a are now

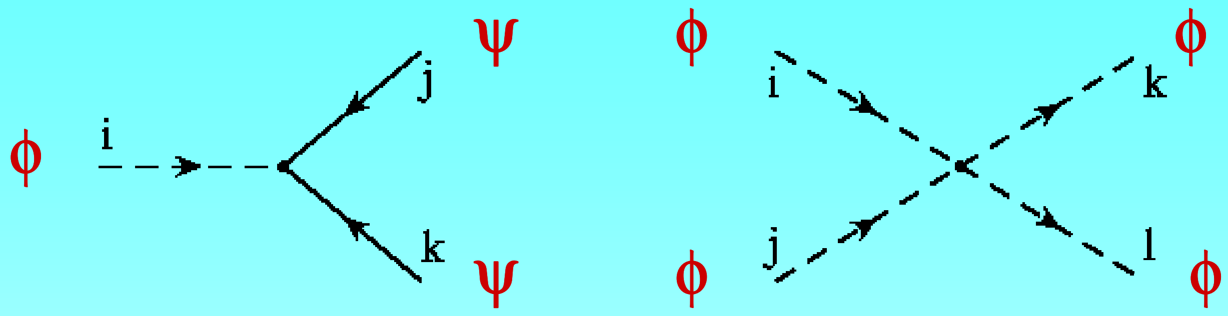
$$D^a = -g(\phi^* T^a \phi)$$

and the scalar potential becomes

$$V(\phi) = F_i^* F_i + \frac{1}{2} D^a D^a = W_i^* W_i + \frac{1}{2} g^2 (\phi^* T^a \phi)(\phi^* T^a \phi)$$

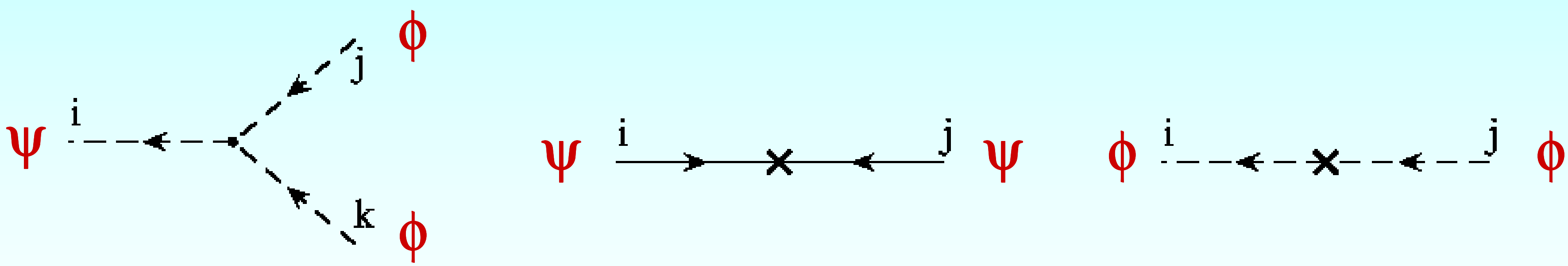
completely determined by the auxiliary fields. This fact is relevant for the discussion of spontaneous supersymmetry breaking.

Couplings in a supersymmetric gauge theory



(a) (b)

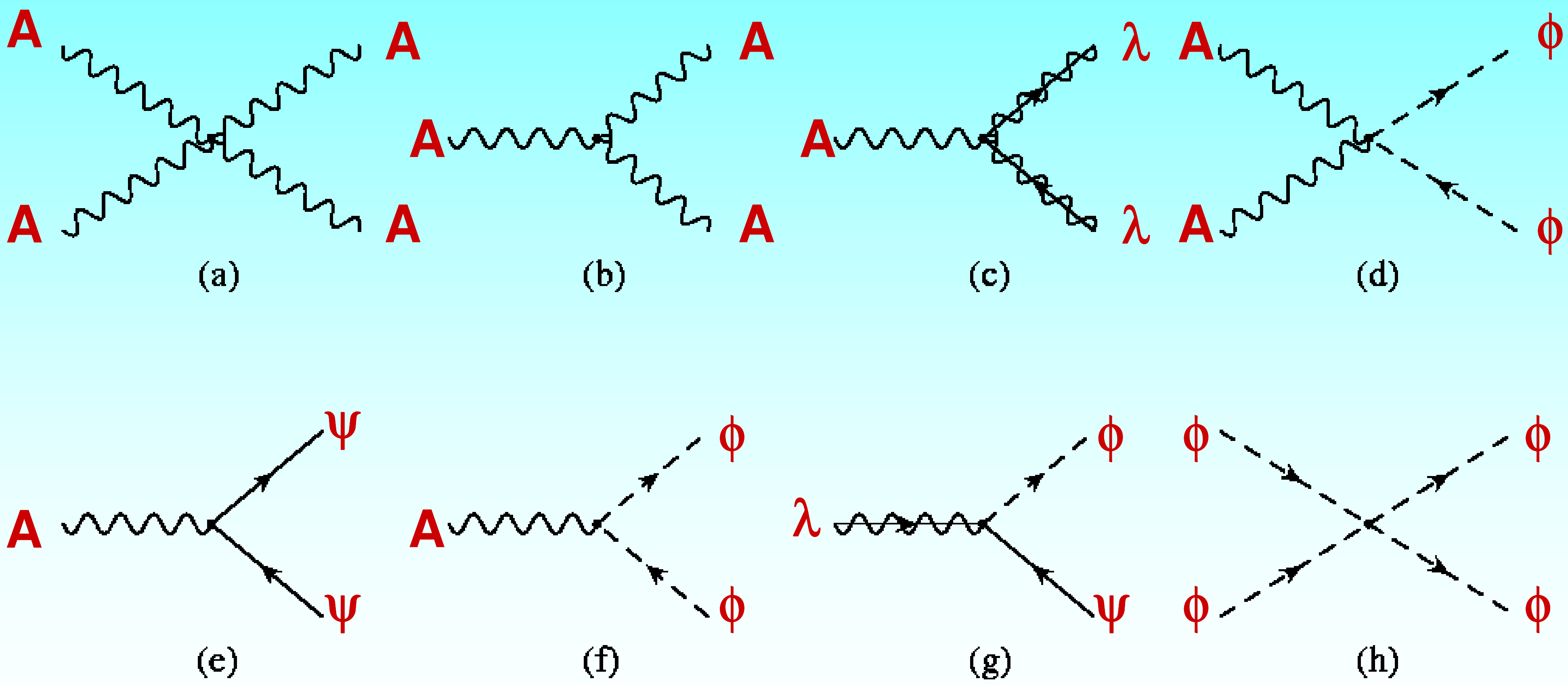
y_{ijk} $y_{ijm}y^*_{klm}$



(a) (b) (c)

$M^*_{im}y_{jkm}$ M_{ij} $M^*_{ik}M_{kj}$

Gauge couplings



Mass terms

Soft supersymmetry breaking

The most general soft supersymmetry breaking lagrangian is given by

$$\mathcal{L}_{\mathrm{soft}} = -(m^2)_{ij}\phi_i\phi_j^* - \left(\frac{1}{2} m_\lambda \lambda^a\lambda^a + \frac{1}{2} b_{ij}\phi_i\phi_j + \frac{1}{6} a_{ijk}\phi_i\phi_j\phi_k + \mathrm{h.c.}\right)$$

It can be shown that a theory with exact supersymmetry, plus $\mathcal{L}_{\mathrm{soft}}$, is free of quadratic divergences.

In principle, there could also be

$$-\frac{1}{2} c_{ijk}\phi_i^*\phi_j\phi_k + \mathrm{h.c.}$$

Usually negligible in most supersymmetry breaking scenarios.

The minimal supersymmetric Standard Model

With some specifications, there is a unique superpotential which is allowed by the Standard Model gauge invariance and by renormalizability:

$$W_{\mathrm{MSSM}} = \tilde{u}^*_R\, y_u\, \tilde{q}_L H_u - \tilde{d}^*_R\, y_d\, \tilde{q}_L H_d - \tilde{e}^*_R\, y_l\, \tilde{\ell}_L H_d + \mu\, H_u\, H_d$$

(call it W from now on). The first term in detail:

$$\tilde{u}^*_R\, y_u\, \tilde{q}_L H_u = (\tilde{u}^*_R)_j{}^f\, (y_u)_{fg}\, (\tilde{q}_L)^{gj}_\alpha\, (H_u)_\beta\, \epsilon^{\alpha\beta}$$

j **is an** $SU(3)_{\mathrm{color}}$ **index,** $j = 1, 2, 3$

α, β **are** $SU(2)_{\mathrm{L}}$ **indices,** $\alpha = 1, 2, \beta = 1, 2$

f, g **are generation indices,** $f = 1, 2, 3,\ g = 1, 2, 3$

If ϕ_1 and ϕ_2 are $SU(2)$ doublets,

$$\delta\phi_1 = i\alpha^a \sigma_a \, \phi_1 \qquad \delta\phi_2 = i\alpha^a \sigma_a \, \phi_2$$

then

$$\phi_1^T \, \epsilon \, \phi_2, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \equiv i\sigma_2$$

is an $SU(2)$ scalar:

$$\delta(\phi_1^T \, \epsilon \, \phi_2) = i\alpha^a \, \phi_1^T \, (\sigma_a^T \epsilon + \epsilon\sigma_a) \, \phi_2 = 0$$

because σ_1, σ_3 are symmetric, σ_2 antisymmetric, and $\{\sigma_a, \sigma_b\} = 2\delta_{ab}$. The quantities

$$\phi_1^\dagger \phi_1 \quad \phi_1^\dagger \phi_2 \quad \phi_2^\dagger \phi_2$$

are also, obviously, $SU(2)$ invariants, by they cannot enter the superpotential, which is an analytic function of all scalar fields.

Some comments:

- **It is now clear why we need two Higgs doublets: in the Standard Model, H gives mass to up-type quarks, while $H^c = \epsilon H^*$ gives mass to down quarks and lepons. Here, H and H^* cannot appear simultaneously in the superpotential.**
- y_u, y_d, y_l **are 3×3 matrices in family space of dimensionless Yukawa couplings. They contain fermion masses and generation mixing. A good approximation:** $y_u^{33} = y_t, y_b^{33} = y_b, y_l^{33} = y_\tau$, **all other entries equal to 0.**
- **No bilinear term other than $\mu H_u H_d$ is allowed by gauge symmetry. μ is the only dimensionful parameter in the superpotential; in particular, mass terms for fermions are forbidden.**

The superpotential originates a large variety of interaction vertices, with coupling constants related by supersymmetry.

Recall our recipe,

$$\mathcal{L}_{\rm ng} = -W_i\, W_i^* - \frac{1}{2}\, W_{ij}\, \psi_i \psi_j - \frac{1}{2}\, W_{ij}^*\, \psi_i^\dagger \psi_j^\dagger$$

where

$$W_i = \frac{\partial W}{\partial \phi_i}, W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

We find e.g.

- Yukawa couplings, such as

$$\frac{\partial W}{\partial \tilde{u}_R^* \partial \tilde{u}_L} \bar{u} u = \bar{u}\, y_u\, u\, H_u^0$$

as in the Standard Model.

- quark-squark-higgsino couplings,

$$\frac{\partial W}{\partial \tilde{u}_R^* \partial H_u^0} \bar{u} \tilde{h}_u^0 = \bar{u}\, y_u\, \tilde{q}_L\, h_u^0$$

- Quartic scalar couplings:

$$\left| \frac{\partial W}{\partial \tilde{u}_R^*} \right|^2 = |y_u\, \tilde{q}_L\, H_u|^2$$

The term $\mu H_u H_d$ provides a higgsino mass term

$$-\mu\,(\tilde{h}_u^+ \tilde{h}_d^- - \tilde{h}_u^0 \tilde{h}_d^0 + \text{h.c.})$$

and a scalar Higgs mass term

$$-\,|\mu|^2 \left(\left|H_u^+\right|^2 + \left|H_u^0\right|^2 + \left|H_d^0\right|^2 + \left|H_d^-\right|^2 \right)$$

This term cannot induce spontaneous breaking of the electroweak gauge symmetry. We must rely upon soft breaking terms.

The value of μ is a problem: we expect it to be of the same order (~ 100 GeV) as the soft breaking terms, in order to provide the correct value of the W and Z masses, but it has a completely different origin.

Accidental symmetries and R-parity

Baryon and lepton number conservation in the Standard Model follow from accidental symmetries: a consequence of renormalizability.

This is no longer true in supersymmetry:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk}\,\tilde{\ell}^i_L\tilde{\ell}^j_L(\tilde{e}^k_R)^* + \frac{1}{2}\lambda'_{ijk}\,\tilde{\ell}^i_L\tilde{q}^j_L(\tilde{d}^k_R)^* + \mu'_i\,\tilde{\ell}^i_L H_u$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk}\,(\tilde{u}^i_R)^*(\tilde{d}^j_R)^*(\tilde{d}^k_R)^*$$

are allowed by renormalizability and gauge symmetry.

In order to implement B and L conservation we must impose some extra symmetry (not B and L themselves: they are violated by nonperturbative effects!).

Consider the so-called matter parity:

$$P_M = (-1)^{3(B-L)}$$

where $B = 1/3$ for quarks and squarks and 0 otherwise, $L = 1$ for leptons and sleptons, 0 otherwise.

P_M commutes with supersymmetry: all particles in the same supermultiplet have the same P_M (-1 for quarks and leptons, $+1$ for Higgs and gauge).

Requiring $P_M = +1$ for all terms in the lagrangian (or in the superpotential) eliminates B and L violating terms.

A different, but equivalent language: define $R-$parity as

$$P_R = (-1)^{3(B-L)+2s}$$

Conserved if P_M is conserved, but does not commute with supersymmetry:

$$P_R = +1 \quad \text{for standard particles}$$
$$P_R = -1 \quad \text{for superpartners}$$

R-parity conserved $\Leftrightarrow$ matter parity conserved.

Useful for phenomenology: each vertex must contain an even number of fields with $P_R = -1$.

Consequences of R-parity conservation

- The lightest particle with $P_R = -1$ is stable. It is called the **LSP**. A candidate for dark matter.
- Supersymmetric particles decay into states with an odd number of $P_R = -1$ particles.
- Supersymmetric particles are always produced in even numbers (usually 2).

Soft supersymmetry breaking in the MSSM

The most general soft supersymmetry breaking term for the MSSM is

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{h.c.}\right)\\ & -\left(\tilde{u}_R^*\, A_u\, \tilde{q}_L\, H_u - \tilde{d}_R^*\, A_d\, \tilde{q}_L\, H_d - \tilde{e}_R^*\, A_e\, \tilde{\ell}_L\, H_d\right) + \text{h.c.}\\ & -\tilde{q}_L^\dagger\, m_Q^2\, \tilde{q}_L - \tilde{\ell}_L^\dagger\, m_L^2\, \tilde{\ell}_L - \tilde{u}_R^\dagger\, m_U^2\, \tilde{u}_R - \tilde{d}_R^\dagger\, m_D^2\, \tilde{d}_R - \tilde{e}_R^\dagger\, m_E^2\, \tilde{e}_R\\ & -m_{H_u}^2\, H_u^\dagger\, H_u - m_{H_d}^2\, H_d^\dagger\, H_d - (B\, H_u\, H_d + \text{h.c.})\end{aligned}$$

An enormous number of new independent parameters.

All of them are expected to be of order m_{soft} (to the appropriate power), something between 100 GeV and 1 TeV.

Soft breaking terms arise after spontaneous breaking of supersymmetry has taken place; we postpone an analysis of this problem.

An anticipation: **two main scenarios have been proposed:**

- **gravity-mediated supersymmetry breaking**
- **gauge-mediated supersymmetry breaking**

The resulting spectrum and couplings (and experimental strategies) are quite different in the two cases.

Do we have any guiding principle that can help us reducing the number of arbitrary parameters?

$m_Q^2, m_U^2, m_D^2, m_E^2$ **are arbitrary 3×3 matrices in flavour space; they are subject from strong contraints from**

- **flavour changing neutral current phenomena (e.g. $K_0 \bar{K}_0$ mixing)**
- **individual leptonic number conservations (e.g. $\mu \to e\gamma$)**
- **CP violation**

The (unknown) mechanism that generates soft breaking terms must respect these constraints.

All these constraints are satisfied with the simple assumption that flavour mixing and CP violation are only originated by Yukawa couplings. This is achieved by assuming that

- $(m_Q^2)_{fg} = m_Q^2 \delta_{fg} \quad (m_U^2)_{fg} = m_U^2 \delta_{fg} \quad (m_D^2)_{fg} = m_D^2 \delta_{fg}$
  $(m_E^2)_{fg} = m_E^2 \delta_{fg}$
- $(A_u)_{fg} = A_u (y_u)^{fg} \quad (A_d)_{fg} = A_d (y_d)^{fg} \quad (A_e)_{fg} = A_e (y_e)^{fg}$
- **all soft breaking parameters are real.**

This (or any other) structure of soft breaking terms results from some underlying mechanism that breaks supersymmetry at a scale Q_0, presumably very large (e.g. the grand-unification scale).

Predictions at energy scales relevant for our experiments, m_{exp}, typically of order 100 to 1000 GeV, will therefore contain large logarithms

$$\log \frac{Q_0}{m_{\text{exp}}}$$

induced by radiative corrections. These must be resummed by standard renormalization group techniques.

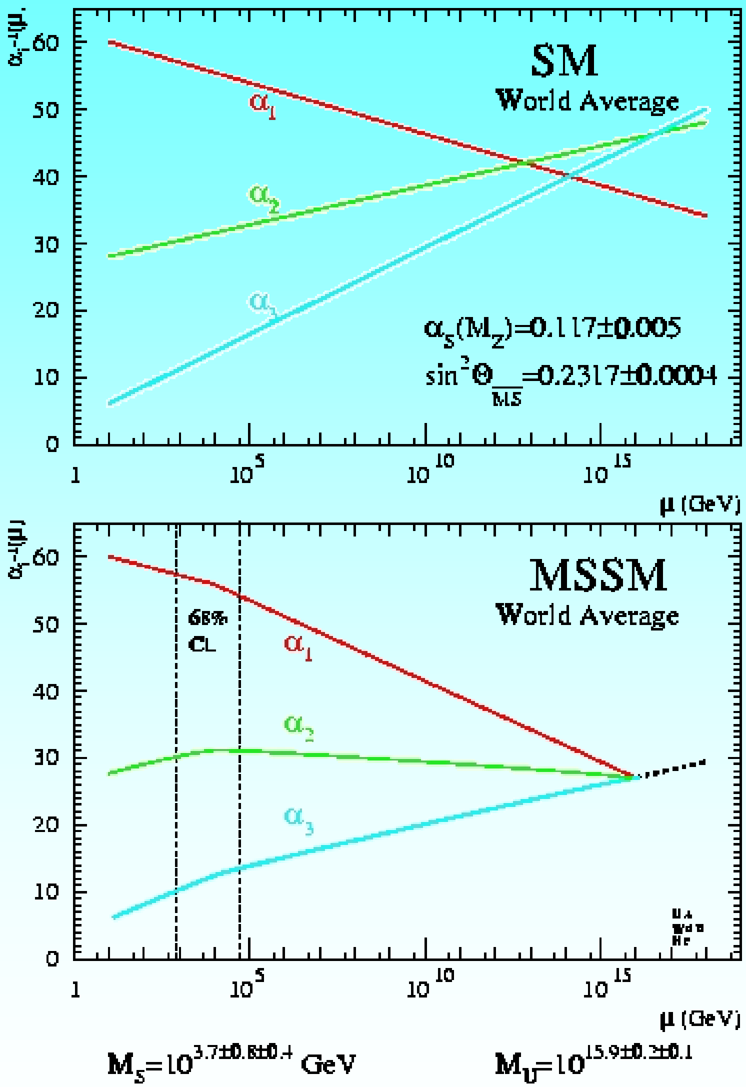
An encouraging fact: coupling constant unification works much better in the MSSM than in the Standar Model. Define

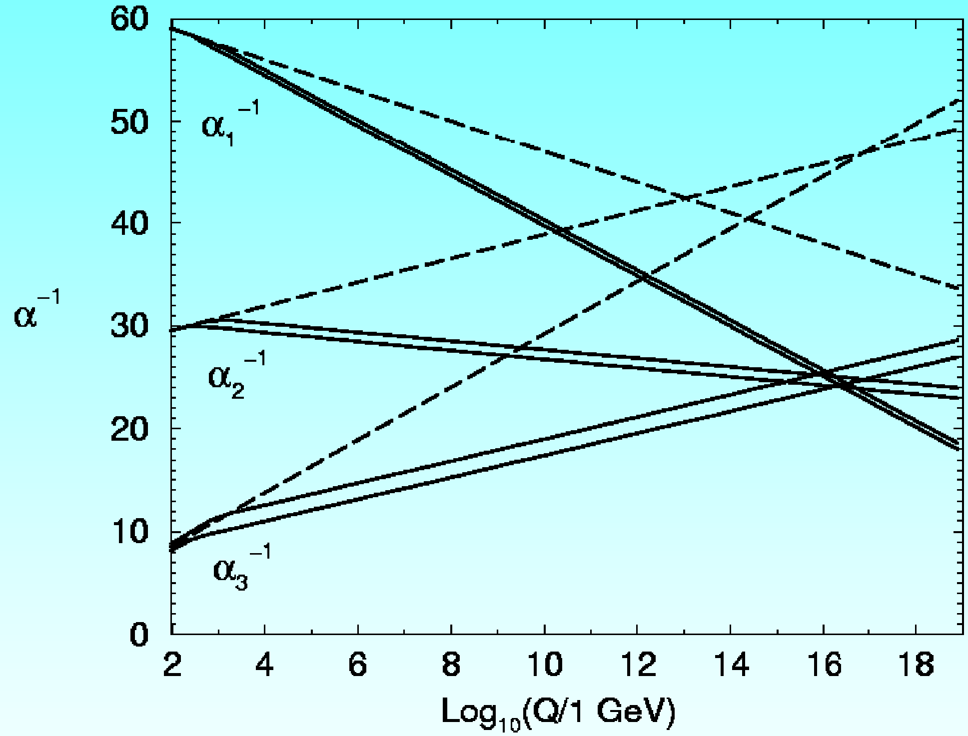
$$g_3 = g_s$$
$$g_2 = g = \frac{e}{\sin\theta_W}$$
$$g_1 = \sqrt{\frac{5}{3}}g' = \sqrt{\frac{5}{3}}\frac{e}{\cos\theta_W}$$

(in order to have the same normalization for symmetry generators). The corresponding RGEs at one loop are

$$\mu^2\frac{dg_i}{d\mu^2} = \frac{b_i}{8\pi}g_i^3 \quad \Rightarrow \quad \mu^2\frac{d}{d\mu^2}\frac{1}{\alpha_i} = -b_i$$

with $\alpha_i = g_i^2/(4\pi)$.


SM
World Average
$\alpha_s(M_Z)=0.117\pm0.005$
$\sin^2\Theta_{\overline{MS}}=0.2317\pm0.0004$
MSSM
World Average
68% CL
μ (GeV)
$M_S=10^{3.7\pm0.8\pm0.4}$ GeV
$M_U=10^{15.9\pm0.2\pm0.1}$


α_1^{-1}
α_2^{-1}
α_3^{-1}
α^{-1}
$\mathrm{Log}_{10}(Q/1\ \mathrm{GeV})$

The mass spectrum

1. The Higgs sector

The Higgs potential of the MSSM is

$$\begin{aligned} V_{\text{Higgs}} &= (|\mu|^2 + m^2_{H_d})(|H^0_d|^2 + |H^-_d|^2) + (|\mu|^2 + m^2_{H_u})(|H^0_u|^2 + |H^+_u|^2) \\ &+B\left(H^+_u H^-_d - H^0_u H^0_d\right) + \text{h.c.} \\ &+\frac{1}{8}(g^2 + g'^2)(|H^0_u|^2 + |H^+_u|^2 - |H^0_d|^2 - |H^-_d|^2)^2 \\ &+\frac{1}{2}g^2|H^+_u H^{0*}_d + H^0_u H^{-*}_d|^2 \end{aligned}$$

No arbitrary quartic couplings: they are fixed by the D terms.

Minimization: **We can take $H_u^+ = H_d^- = 0$ (good news: EM unbroken) and H_u^0, H_d^0 real and positive at the minimum without loss of generality. Then we restrict to the neutral sector:**

$$V_{\text{Higgs}} = (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 + (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 - (BH_u^0 H_d^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

B may also be taken to be real and positive: no CP violating phases in the Higgs sector.

Perform an $SU(2)_L$ gauge rotation that makes $\langle H_u^+ \rangle = 0$; then, the condition $\partial V / \partial H_u^+ = 0$ is only fulfilled if also $\langle H_d^- \rangle = 0$.

The only place where the phases of H_u^0, H_d^0 appear is the term

$$-B H_u^0 H_d^0 + \text{h.c.}$$

We can make B real and positive by a suitable phase choice for H_u^0, H_d^0.

Clearly, the potential has a minimum if also $H_u^0 H_d^0$ is real and positive, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases, which can be rotated away by a $U(1)_Y$ gauge transformation.

Conditions for electroweak spontaneous symmetry breaking:

1. **The origin $H_u^0 = H_d^0 = 0$ is not a minimum of the potential**
2. **The potential is bounded from below**

Condition 1. leads to the constraint

$$(|\mu|^2 + m_{H_d}^2)(|\mu|^2 + m_{H_u}^2) < B^2$$

In some cases, guaranteed by large negative radiative corrections to $m_{H_u}^2$.

Condition 2. is almost automatic: the quartic couplings are positive definite. There are however D-flat directions $\left|H_u^0\right| = \left|H_d^0\right|$. We must require that the potential in these directions is stabilized by quadratic terms:

$$2B < 2\left|\mu\right|^2 + m_{H_d}^2 + m_{H_u}^2$$

Define $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$. We find

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_u^2 + v_d^2) \Rightarrow v^2 \equiv v_u^2 + v_d^2 \simeq (174\,\mathrm{GeV})^2$$

Following tradition, we also define an angle β through

$$\tan\beta = \frac{v_u}{v_d}$$

Clearly $0 \leq \beta \leq \pi/2$. The minimization conditions are

$$|\mu|^2 + m_{H_d}^2 = B\tan\beta - \frac{m_Z^2}{2}\cos 2\beta$$

$$|\mu|^2 + m_{H_u}^2 = B\cot\beta + \frac{m_Z^2}{2}\cos 2\beta$$

The μ problem is now evident.

Mass eigenstates:

1. Two CP-odd neutral scalars

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\, H_u^0 \\ \mathrm{Im}\, H_d^0 \end{pmatrix}$$

2. Two charged scalars

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^+ \end{pmatrix}$$

$G^0, G^\pm$ are pseudo-Goldstone bosons: they can be removed from the spectrum by an appropriate gauge transformation.

$A^0, H^\pm$ are physical degrees of freedom.

3. Two CP-even neutral scalars

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}\, H_u^0 - v_u \\ \mathrm{Re}\, H_d^0 - v_d \end{pmatrix}$$

Finding the mass eigenvalues is a matter of algebra:

$$m_A^2 = \frac{2B}{\sin 2\beta}$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2$$

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\right)$$

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2} \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}$$

It is easy to show that

$$m_h < |\cos 2\beta|\, m_Z$$

Radiative corrections weaken this bound. At one loop, and neglectic squark mixing, one finds

$$\Delta m_h^2 \simeq \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\log\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$$

An extra term appears if mixing is taken into account:

$$(\Delta m_h^2)_{\rm mix} \simeq \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\frac{(A_t-\mu\cot\beta)^2}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\left[\log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}+(A_t-\mu\cot\beta)^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\right]$$

$$f(a,b)=\frac{1}{a-b}\left[1-\frac{1}{2}\frac{a+b}{a-b}\log\frac{a}{b}\right]$$

Including all corrections, one obtains approximately

$$m_h \lesssim 130 \text{ GeV}$$

still very interesting in view of LHC.

Yukawa couplings are relates to fermion masses in the third generation by

$$y_t = \frac{m_t}{\sqrt{2} m_W \sin\beta} \qquad y_b = \frac{m_b}{\sqrt{2} m_W \cos\beta} \qquad y_\tau = \frac{m_\tau}{\sqrt{2} m_W \cos\beta}$$

A difference with respect to the Standard Model: y_b and y_τ may be as large as y_t, if $\tan\beta$ is large enough ($\tan\beta < 1$ strongly disfavoured).

The couplings of h, H, A to standard particles are the same as in the Standard Model, rescaled by $\alpha-$ and $\beta-$dependent factors:

	$d\bar{d}, s\bar{s}, b\bar{b}$ $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$	$u\bar{u}, c\bar{c}, t\bar{t}$	W^+W^-, ZZ
h	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$\sin(\beta-\alpha)$
H	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$	$\cos(\beta-\alpha)$
A	$-i\gamma_5\tan\beta$	$-i\gamma_5\cot\beta$	0

At tree level, the Higgs sector is described by two parameters (a common choice: m_A^2 and $\tan\beta$).

2. Neutralinos and charginos

The supersymmetric partner of neutral gauge bosons, $\tilde{w}^0$ and $\tilde{b}$, and of Higgs scalars, $\tilde{h}_u^0, \tilde{h}_d^0$, are not mass eigenstates. We find

$$\mathcal{L}_n = -\frac{1}{2}\,\tilde{\psi}_0^T\, M_n\, \tilde{\psi}_0 + \text{h.c.}$$

where the matrix M_n in the basis $\tilde{b}, \tilde{w}^0, \tilde{h}_d^0, \tilde{h}_u^0$ is given by

$$\begin{bmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_W & m_Z \sin\beta \sin\theta_W \\ 0 & M_2 & m_Z \cos\beta \cos\theta_W & -m_Z \sin\beta \cos\theta_W \\ -m_Z \cos\beta \sin\theta_W & m_Z \cos\beta \cos\theta_W & 0 & -\mu \\ m_Z \sin\beta \sin\theta_W & -m_Z \sin\beta \cos\theta_W & -\mu & 0 \end{bmatrix}$$

The mass matrix M_n is symmetric, and can be diagonalized by a unitary transformation N. The four mass eigenstates

$$\tilde{\chi}_i^0 = N_{ij}\,\tilde{\psi}_j^0; \quad M_n = N^T\,\hat{M}_n\,N$$

are called neutralinos. Their coupling are entirely determined by the matrix N, and thus by the four parameters

$$M_1, M_2, \mu, \tan\beta$$

The lightest neutralino is in most models the LSP; neutralinos play a crucial role in phenomenology.

Charged gauginos and higgsinos

$$\tilde{w}^+, \tilde{h}_u^+, \tilde{w}^-, \tilde{h}_d^-$$

mix in a similar way:

$$\mathcal{L}_c = -\frac{1}{2}\,\tilde{\psi}_c^T\, M_c\, \tilde{\psi}_c + \mathrm{h.c.}$$

$$M_c = \begin{bmatrix} 0 & X^T \\ X & 0 \end{bmatrix} \qquad X = \begin{bmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{bmatrix}$$

The mass eigenstates (called charginos) are found by diagonalizing X with a bi-unitary transformation:

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = V \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix} \qquad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = U \begin{pmatrix} \tilde{w}^- \\ \tilde{h}_d^- \end{pmatrix}$$

$$X = U^T \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} V$$

$$m^2_{\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} = \frac{1}{2}\Big[\left(|M_2|^2 + |\mu|^2 + 2m_W^2\right) \mp \sqrt{\left(|M_2|^2 + |\mu|^2 + 2m_W^2\right)^2 - 4\left|\mu M_2 - m_W^2 \sin 2\beta\right|^2} \Big]$$

As in the case of neutralinos, the matrices U, V contain all the relevant information about chargino couplings.

It is quite natural to assume that gaugino masses have a common value $m_{1/2}$ at the reference scale Q_0, where the (properly normalized) gauge couplings take approximately the same value $\alpha_U \sim 0.04$.

It turns out that gaugino running masses obey the same renormalization group equations. Therefore, with the above assumption,

$$M_i(Q) = \frac{\alpha_i(Q)}{\alpha_U} m_{1/2}$$

This reduces the number of arbitrary parameters in the gaugino sector.

3. Squarks and sleptons

The scalar partners of matter fermions (squarks and sleptons) have mass matrices with contributions from various terms:

- **quartic Higgs-Higgs-squark-squark vertices**
- **trilinear Higgs-squark-squark couplings**
- **soft-breaking masses**
- D**-terms**

In general, there is mixing among different families, and mixing betweel scalar parners of left- and right-handed fermions. This can be sizeable in the third generation.

Soft supersymmetry breaking in the MSSM

The most general soft supersymmetry breaking term for the MSSM is

$$
\begin{aligned}
\mathcal{L}_{\rm soft} \;=\; & -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{h.c.}\right) \\
& -\left(\tilde{u}_R^* \, A_u \, \tilde{q}_L \, H_u - \tilde{d}_R^* \, A_d \, \tilde{q}_L \, H_d - \tilde{e}_R^* \, A_e \, \tilde{\ell}_L \, H_d\right) + \text{h.c.} \\
& -\tilde{q}_L^\dagger \, m_Q^2 \, \tilde{q}_L - \tilde{\ell}_L^\dagger \, m_L^2 \, \tilde{\ell}_L - \tilde{u}_R^\dagger \, m_U^2 \, \tilde{u}_R - \tilde{d}_R^\dagger \, m_D^2 \, \tilde{d}_R - \tilde{e}_R^\dagger \, m_E^2 \, \tilde{e}_R \\
& -m_{H_u}^2 \, H_u^\dagger \, H_u - m_{H_d}^2 \, H_d^\dagger \, H_d - (B \, H_u \, H_d + \text{h.c.})
\end{aligned}
$$

An enormous number of new independent parameters.

All of them are expected to be of order $m_{\rm soft}$ (to the appropriate power), something between 100 GeV and 1 TeV.

Mass matrices for squarks and sleptons in the third generation, in the simplified situation outlined above:

$$\mathbf{s-top:}\quad (\tilde{t}_L^* \ \ \tilde{t}_R^*) \begin{pmatrix} m_Q^2 + m_t^2 + \Delta_Q & m_t(A_t - \mu \cot\beta) \\ m_t(A_t - \mu \cot\beta) & m_U^2 + m_t^2 + \Delta_U \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$\mathbf{s-bottom:}\quad (\tilde{b}_L^* \ \ \tilde{b}_R^*) \begin{pmatrix} m_Q^2 + m_b^2 + \Delta_Q & m_b(A_b - \mu \tan\beta) \\ m_b(A_b - \mu \tan\beta) & m_D^2 + m_b^2 + \Delta_D \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}$$

$$\mathbf{s-tau:}\quad (\tilde{\tau}_L^* \ \ \tilde{\tau}_R^*) \begin{pmatrix} m_L^2 + m_\tau^2 + \Delta_L & m_\tau(A_\tau - \mu \tan\beta) \\ m_\tau(A_\tau - \mu \tan\beta) & m_E^2 + m_\tau^2 + \Delta_E \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

where

$$\Delta = m_Z^2 (T_3 - Q \sin^2\theta_W) \cos 2\beta$$

Spontaneous supersymmetry breaking

How are soft supersymmetry-breaking terms generated? We need a mechanism of spontaneous supersymmetry breaking (explicit breaking not acceptable).

A symmetry is said to be spontaneously broken when the vacuum state is not invariant under that symmetry:

$$Q|0\rangle \neq 0$$

Supersymmetry is no exception. This can happen only if some of the auxiliary fields F_i or D^a acquires a non-zero vacuum expectation value.

Proof: the supersymmetry algebra implies

$$H = P^0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right)$$

which implies

$$Q_i|0\rangle \neq 0 \;\Leftrightarrow\; \langle 0|H|0\rangle > 0 \;\Leftrightarrow\; \langle 0|V|0\rangle > 0$$

for a translation-invariant vacuum state. But

$$V = F_i F_i^* + \frac{1}{2} D^a D^a$$

so supersymmetry is spontaneously broken if and only if the system

$$\langle 0|F_i|0\rangle = 0 \qquad \langle 0|D^a|0\rangle = 0$$

has no solution.

Spontaneous breaking based on $\langle D^a \rangle \neq 0$ for some D^a is called à la Fayet-Iliopoulos. If there is a $U(1)$ factor in the gauge group, a term

$$\mathcal{L}_{\mathrm{D-breaking}} = kD$$

can be added to the lagrangian. Then

$$V = \frac{1}{2}D^2 - kD + gD\sum_i q_i \phi_i \phi_i^*$$

$$D = k - g\sum_i q_i \phi_i \phi_i^*$$

D is different from zero if for some reason all scalar fields are zero at the minimum. This is *not* the case in the MSSM.

Not very promising, although not completely ruled out.

Models in which supersymmetry is spontaneously broken by $\langle F_i \rangle \neq 0$ for some F_i are called O'Raifeartaigh models. An explicit example: three chiral superfields, and

$$W = -k\phi_1 + m\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

where ϕ_1 must be a gauge singlet. Then

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2} \quad F_2 = -m\phi_3^* \quad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*$$

$F_1 = F_2 = F_3 = 0$ has no solution. For $m^2 > yk$, the minimum of V is for $\phi_2 = \phi_3 = 0$, ϕ_1 undetermined (flat direction).

The solution is $F_1 = k$, $V = k^2$ at the minimum. The flat direction is no longer flat after radiative corrections, and the minimum is at $\phi_1 = 0$. The spectrum: six scalars with masses

$$0 \quad 0 \quad m^2 \quad m^2 \quad m^2 - yk \quad m^2 + yk$$

and three fermions with masses

$$0 \quad m \quad m$$

The zero-mass fermion is the analog of a Goldstone boson in broken ordinary symmetry: we call it a goldstino (sic). It is the partner of the auxiliary field F_1, responsible for supersymmetry breaking.

A general conclusion: spontaneous breaking of supersymmetry cannot take place within the MSSM. There is no gauge-singlet supermultiplet whose auxiliary field can take a non-vanishing vacuum expectation value.

Hard to do by simply extending the MSSM with new fields and new renormalizable interactions:

- **gaugino masses cannot arise from renormalizable couplings**
- **the spectrum must include light scalars:**

$$\mathrm{STr}\mathcal{M}^2 = \sum_i (-1)^{(2s)}(2s+1)m_i^2 = 0$$

even after supersymmetry breaking.

Mass terms are generated after spontaneous symmetry breaking as, for example, quark masses in the standard model:

$$y\,\phi\bar{\psi}\psi \;\rightarrow\; y\,\langle\phi\rangle\bar{\psi}\psi = m\bar{\psi}\psi$$

In supersymmetry, the same mechanism is at work. A gaugino mass could be generated either by a coupling

$$\phi\lambda\lambda \rightarrow \langle\phi\rangle\lambda\lambda$$

but such a term is absent in a supersymmetric theory, or by

$$F\lambda\lambda \rightarrow \langle F\rangle\lambda\lambda$$

which is not renormalizable.

Way out: assume that supersymmetry is broken in a hidden sector of non-observable degrees of freedom, that communicate somehow with the observable sector. Historically, two proposals: supersymmetry breaking communicated to the observable sector by

1. gravitational interactions
2. gauge interactions

The differences between the two cases are due to a different size of the supersymmetry breaking scale

$$\langle 0|F|0\rangle$$

In gravity mediated supersymmetry breaking, soft breaking terms arise from (in general nonrenormalizable) interaction terms (of gravitational strength) of observable fields with combinations of fields in the hidden sector that acquire a non-zero vev. We expect therefore

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}$$

on dimensional grounds: m_{soft} must vanish as $\langle F \rangle$ goes to zero (no spontaneous breaking) and as $M_P \to \infty$ (no interaction between hidden and observable sectors.

For $m_{\text{soft}} \sim 1$ TeV we get

$$\sqrt{\langle F \rangle} \sim 10^{11}\ \text{GeV}$$

In the gauge mediated scenario, superymmetry breaking is communicated to the observable sector by the ordinary gauge interactions via loop effects due to particles called messengers.

A rough estimate of the relevant supersymmetry breaking scale can be obtained. We have in this case

$$m_{\mathrm{soft}} \sim \frac{\alpha}{4\pi}\frac{\langle F\rangle}{M}$$

where M is the typical messenger mass. This corresponds to a much lower value for the supersymmetry order parameter:

$$\sqrt{\langle F\rangle} \sim 10^4 - 10^5 \ \mathrm{GeV}$$

if $\langle F\rangle$ and M are assumed to be roughly of the same size.

A closer look to the goldstino

The goldstino field can be easily identified. The full fermion mass matrix of a generic supersymmetric theory is

$$M_f = \begin{pmatrix} 0 & \sqrt{2}g_a(T^a\langle\phi\rangle)_i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)_j & \langle W_{ij}\rangle \end{pmatrix}$$

There is always one combination of fermion fields that corresponds to zero mass:

$$\tilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}$$

is an eigenvector of M_f with zero eigenvalue. This is the goldstino.

The first component of $M_f\tilde{G}$ is

$$\sqrt{2}\langle g_a(T^a\phi)_i F_i\rangle = -\sqrt{2}\langle g_a(T^a\phi)_i W_i^*\rangle = -\sqrt{2}\langle \frac{\partial W^*}{\partial \phi_i^*}\delta_{\rm gauge}\phi_i^*\rangle = 0$$

by gauge-invariance of the superpotential.

The second component is given by

$$\langle g_a(\phi^* T^a)_j D^a + W_{ij}F_i\rangle$$

The scalar potential is

$$V = F_i F_i^* + \frac{1}{2}D^a D^a = W_i W_i^* + \frac{1}{2}g_a^2(\phi^* T^a \phi)^2$$

It follows that

$$\begin{aligned}\langle\frac{\partial V}{\partial \phi_k}\rangle &= \langle W_i^* W_{ik} + g_a^2(\phi^* T^a\phi)(\phi^* T^a)_k\rangle \\ &= -\langle F_i W_{ik} + g_a D^a(\phi^* T^a)_k\rangle = 0\end{aligned}$$

Let us consider the case of local supersymmetry: $\eta = \eta(x)$. Then

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\eta_1^\dagger\,\bar{\sigma}^\mu\,\eta_2 - \eta_2^\dagger\,\bar{\sigma}^\mu\,\eta_1)\,i\partial_\mu X$$

is a local translation, i.e. a general coordinate transformation.

Gravitation is included in a natural way.

Local supersymmetry is called supergravity.

In this case, the so-called super-Higgs effect takes place: the spin-3/2 partner of the graviton, the gravitino, acquires a mass.

The missing spin degrees of freedom are provided by the goldstino.

By dimensional analysis,

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P}$$

Clearly, the gravitino mass has very different values in gravity-mediated and gauge mediated scenarios:

- **gravity-mediated: the gravitino mass**

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim m_{\rm soft}$$

and its interactions are gravitational. Almost no interest for phenomenology at colliders.

- **gauge-mediated:**

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim \frac{10^5}{10^{19}}\ {\rm GeV}$$

if $M \sim \langle F \rangle \ll M_P$. Its goldstino components may have non-negligible couplings, and may therefore play a role in LHC physics.

Gravity-mediated models: some details

The supergravity lagrangian is non-renormalizable (as expected). Assume there is a hidden chiral superfeld X whose auxiliary component F_X gets a non-zero vev. The lagrangian includes

$$\begin{aligned}\mathcal{L} &= -\frac{1}{M_P} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} \\ &+ \frac{1}{M_P^2} F_X F_X^* k_{ij} \phi_i^* \phi_j \\ &- \frac{1}{M_P} F_X \left(\frac{1}{6} y'_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'_{ij} \phi_i \phi_j + \text{h.c.} \right)\end{aligned}$$

When $F_X \to \langle F_X \rangle$, this gives precisely the soft terms we need.

Soft supersymmetry breaking in the MSSM

The most general soft supersymmetry breaking term for the MSSM is

$$\begin{aligned} \mathcal{L}_{\text{soft}} &= -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{h.c.}\right) \\ &\quad -\left(\tilde{u}_R^* A_u \tilde{q}_L H_u - \tilde{d}_R^* A_d \tilde{q}_L H_d - \tilde{e}_R^* A_e \tilde{\ell}_L H_d\right) + \text{h.c.} \\ &\quad -\tilde{q}_L^\dagger m_Q^2 \tilde{q}_L - \tilde{\ell}_L^\dagger m_L^2 \tilde{\ell}_L - \tilde{u}_R^\dagger m_U^2 \tilde{u}_R - \tilde{d}_R^\dagger m_D^2 \tilde{d}_R - \tilde{e}_R^\dagger m_E^2 \tilde{e}_R \\ &\quad -m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (B\, H_u H_d + \text{h.c.}) \end{aligned}$$

An enormous number of new independent parameters.

All of them are expected to be of order m_{soft} (to the appropriate power), something between 100 GeV and 1 TeV.

In a *minimal* version of supergravity, there are considerable simplifications:

$$f_a = f \quad k_{ij} = k\delta_{ij} \quad y' = \alpha y \quad \mu' = \beta\mu$$

This leads to

$$M_1 = M_2 = M_3 = m_{1/2}$$
$$m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_{H_u}^2 = m_{H_d}^2 = m_0^2$$
$$A_u = A_0 y_u \quad A_d = A_0 y_d \quad A_e = A_0 y_e$$
$$B = B_0 \mu$$

Neutral flavour changing effects automatically suppressed.

Gauge-mediated models: some details

In the simplest version, the model contains four messenger supermultiplet:

$$q \sim \left(\mathbf{3}, \mathbf{1}, -\frac{2}{3}\right) \quad \bar{q} \sim \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3}\right) \quad \ell \sim (\mathbf{1}, \mathbf{2}, +1) \quad \bar{\ell} \sim (\mathbf{1}, \mathbf{2}, -1)$$

and a chiral multiplet S, whose scalar and auxiliary components take a vev (e.g. by the O'Raifeartaigh mechanism). They interact through a superpotential

$$W_{\text{messengers}} = y_2 S \ell \bar{\ell} + y_3 S q \bar{q}$$

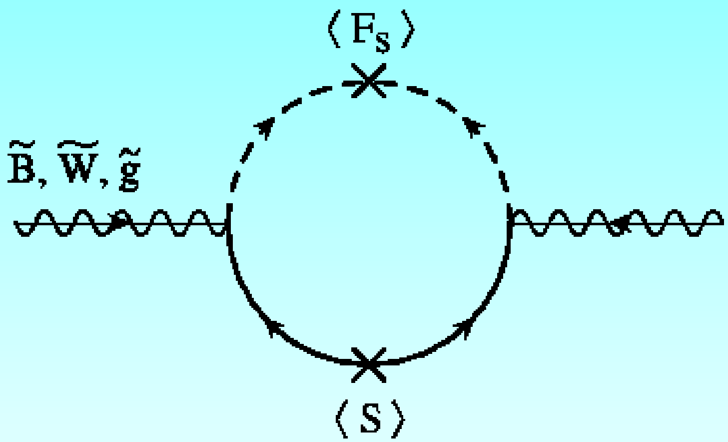
which becomes effectively

$$W_{\text{messengers}} = y_2 \langle S \rangle \ell \bar{\ell} + y_3 \langle S \rangle q \bar{q}$$

Gauginos get masses

$$M_a = \frac{\alpha_a}{4\pi}\frac{\langle F_S\rangle}{\langle S\rangle}$$

through one-loop diagrams:



Soft scalar masses and trilinear couplings arise at two-loops. One gets

$$A_u = A_d = A_e = 0$$

at the messenger scale.

General features of soft masses in gauge-mediated models:

- **larger for strongly-interacting particles**
- **determined by gauge properties only**

The second properties leads to a degeneracy in squarks and slepton masses, that suppresses FCNC effects.

Summary

- Supersymmetry provides a solution to the naturalness problem without spoiling the good features of the Standard Model.
- It has a number of welcome side-effects:
  - a better context for grand unification
  - a natural candidate for cold dark matter.
- A large portion of the parameter space for supersymmetric theories has already been explored.
- Supersymmetry may be relevant in different forms (superstrings).