Beyond the Standard Model

Lecture 1

Leandar Litov
University of Sofia

Introduction

- The Standard Model
- Grand Unification
- SUSY
- Extra Dimensions
- Strings

Fundamental particles

Fundamental particles

LEPTONS

- Do not participate in strong interactions
- Spin ½
- Observed as free particles
- Pointlike (r < few x 10^{-17}cm)

QUARKS

- Strong interactions bind them into hadrons
- Not observed as free particle – confinement
- Spin ½ ; pointlike; (r < few x 10^{-17}cm)
- $Q_u = 2/3$; $Q_d = -1/3$

Family (Generation) Structure

$$\begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix}$$

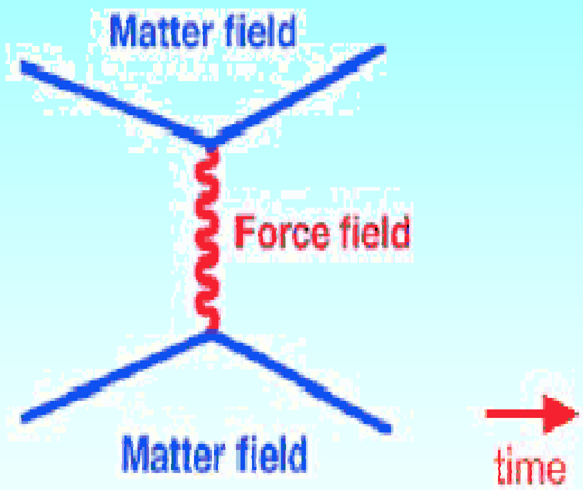
$N_G = 3$

Interactions

Forces are transmitted by the exchange of (force) particles between (matter) particles

Explains the differences between forces
To verify : look for force particles

$$\text{Range of a Force} \propto \frac{1}{\text{mass of exchange particle}}$$



Observe 4 forces
There are 4 different types of force fields

Standard Model - interactions

In QFT – the local invariance of $\mathscr{L}$ defines the interactions

Electromagnetic Interactions: γ

Quantum Electrodynamics (QED) U(1)

In the Standard Model

QED + Weak Interactions: γ , Z^0, $W^\pm$

Electroweak Theory $SU(2)_L \otimes U(1)_Y$

Strong Interaction **8 Gluons**

Quantum Chromodynamics (QCD) $SU_c(3)$

Fundamental particles

Three Families

$$\begin{pmatrix} \nu_e & u \\ e^- & d' \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu^- & s' \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau^- & b' \end{pmatrix}$$

Family Structure

$$\begin{pmatrix} \nu_l & q_u \\ l_j & q_d \end{pmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, (\nu_l)_R, l_R^- \right\}; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, (q_u)_R, (q_d)_R \right\}$$

Charged Currents $W^\pm$

Left-handed fermions only

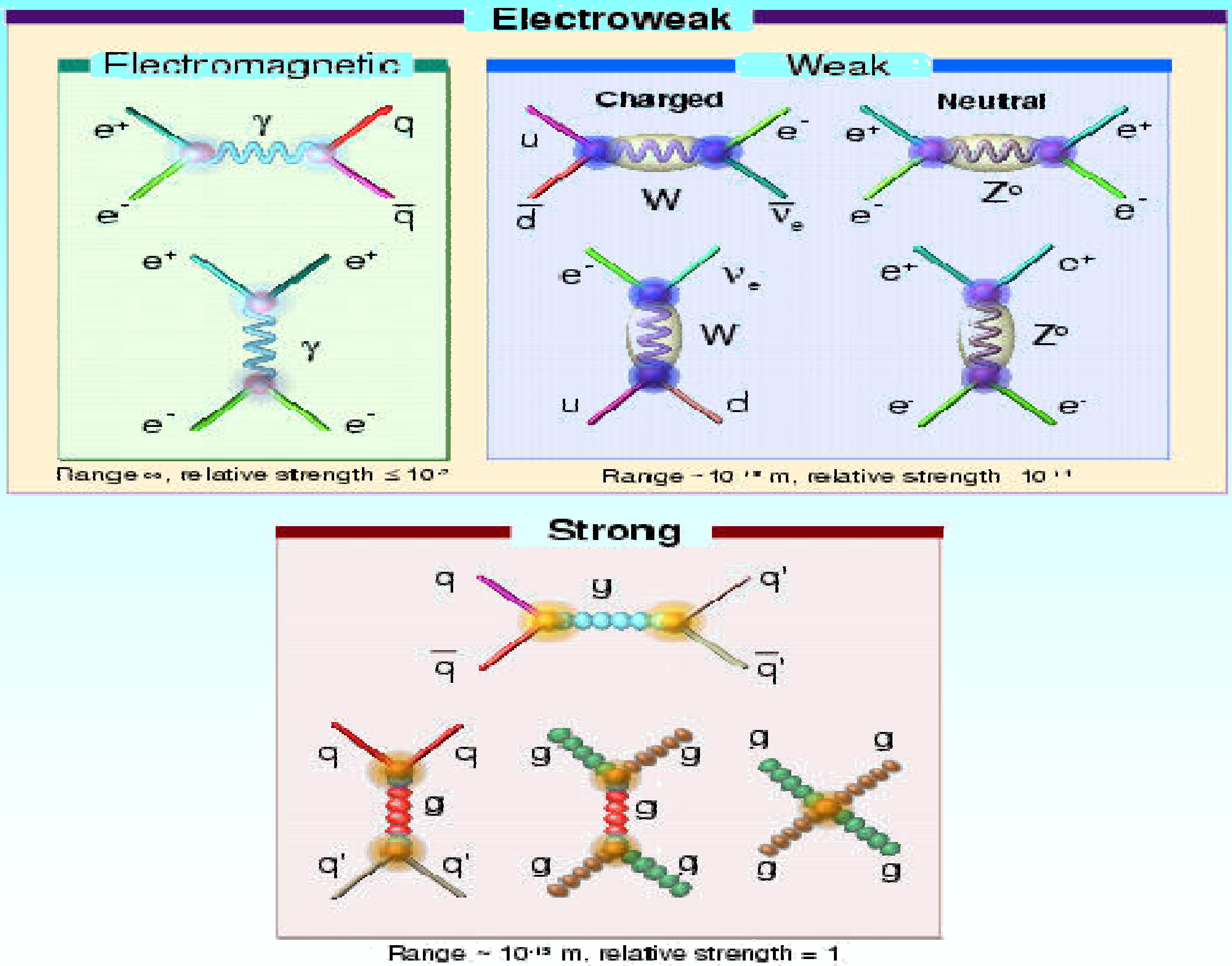
Flavor changing: $\nu_l \Leftrightarrow l$, $q_u \Leftrightarrow q_d$

Neutral Currents γ, Z

Flavor Conserving $f_i \Leftrightarrow f_i$

Standard Model - interactions

Interactions: coupling of forces to matter



Standard Model

PROBLEM WITH MASS SCALES

Gauge Symmetry → $m_\gamma = 0$ Good

$M_W = M_Z = 0$ Bad!

$M_W = 80.43\ \text{GeV}$

$M_Z = 91.19\ \text{GeV}$

Moreover $\mathcal{L}_{m_f} \equiv -m_f\ \bar{f} f = -m_f\ (\bar{f}_L f_R + \bar{f}_R f_L)$

Also Forbidden by Gauge Symmetry → $m_f = 0 \quad \forall f$

All Particles Massless

Spontaneous Symmetry Breaking

In the SM masses are generated trough

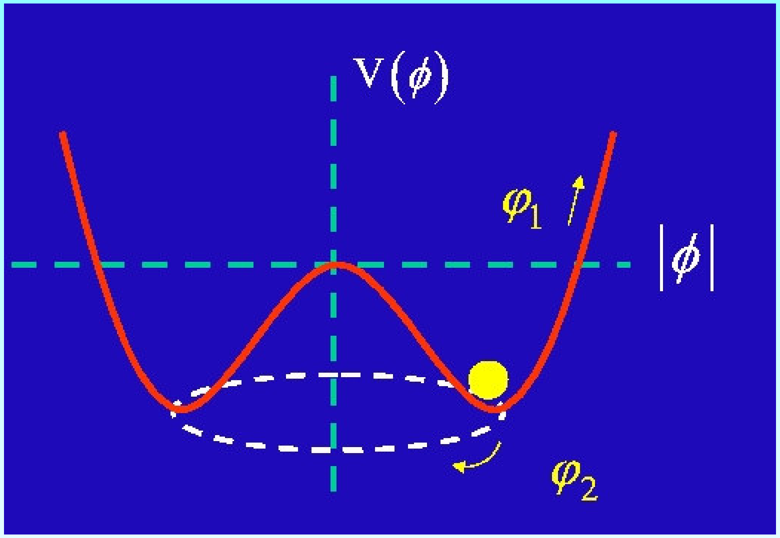
Spontaneous Symmetry Breaking (**SSB**) – Higgs Mechanism

Introduce Scalar Higgs doublet →

The Lagrangian is invariant

However its vacuum state is degenerate –

$$|\langle 0|\Phi_0|0\rangle| = \frac{v}{\sqrt{2}}$$



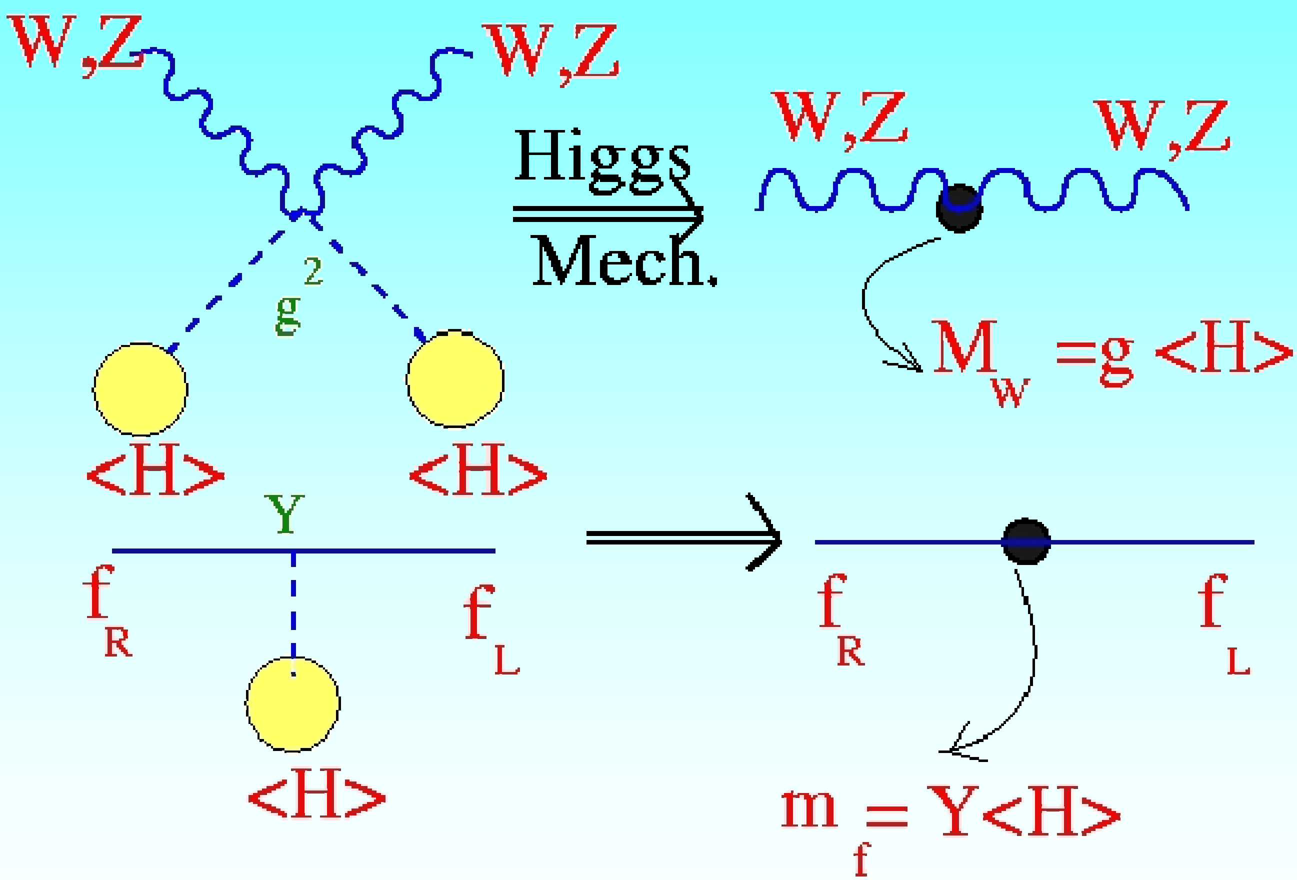
Choice of the vacuum state – leads to SSB

$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$

Couplings with gauge bosons and fermions – induce mass terms

Price – new particle **H-boson** – to be discovered

Higgs interactions



Fermion Masses

$$\begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix}$$

N_G=3 identical copies : f' are massles weak eigenstates

Scalar doublet couples with fermions – allowed by the Gauge Symmetry

SSB

$$L_Y = -\left(1+\frac{H}{v}\right)[\overline{d'_L} M'_d d'_R + \overline{u'_L} M'_u u'_R + \overline{l'_L} M'_l l'_R + h.c.]$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$[M'_d, M'_u, M'_l]_{jk} = -[c^{(d)}_{jk}, c^{(u)}_{jk}, c^{(l)}_{jk}]\frac{v}{\sqrt{2}}$$

Diagonalization of Mass Matrices

$M'_f = S_f^+ M_f S_f U_f$ $\quad S_f^+ S_f = 1$ $\quad U_f^+ U_f = 1$

$$L_Y = -\left(1 + \frac{H}{v}\right)[\overline{d} M_d d + \overline{u} M_u u + \overline{l} M_l l]$$

$M_u = diag(m_u, m_c, m_t)$ $\quad M_d = diag(m_d, m_s, m_b)$ $\quad M_l = diag(m_e, m_\mu, m_\tau)$

$f_L = S_f f'_L$ $\quad f_R = S_f U_f f'_R$

Mass Eigenstates # Weak Eigenstates

$\overline{f'_L} f'_L = \overline{f_L} f_L \quad \overline{f'_R} f'_R = \overline{f_R} f_R \Longrightarrow L'_{NC} = L_{NC}$

$\overline{u'_L} d'_L = \overline{u_L} V d_L$ $\quad V = S_u S_d^+ \Longrightarrow L'_{CC} \neq L_{CC}$

Quark Mixing → V ← **CKM Matrix**

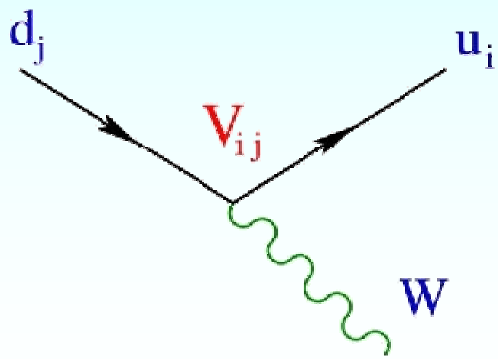
Quark Mixing

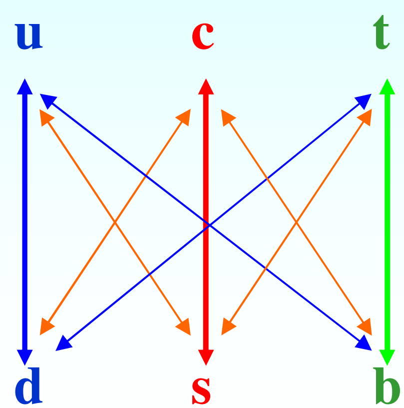
$$L_{NC}^{Z} = \frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \overline{f}\gamma_\mu [v_f - a_f \gamma_5] f$$

Flavour Conserving Neutral Current

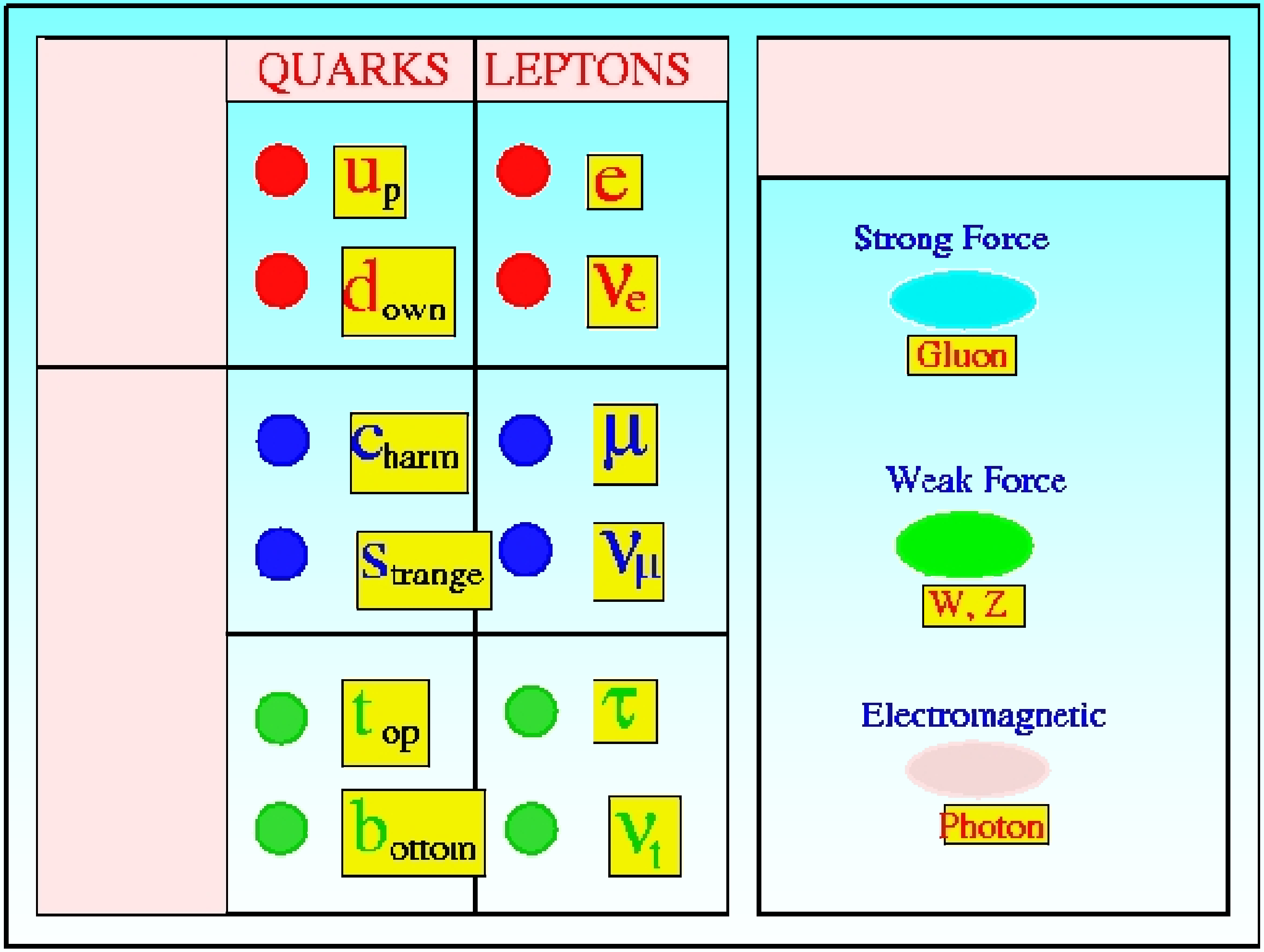
$$L_{CC}^{W} = \frac{g}{2\sqrt{2}} W_\mu^+ [\sum_{ij} \overline{u_i}\gamma^\mu (1-\gamma_5) V_{ij} d_j + \sum_l \overline{\upsilon_l}\gamma^\mu (1-\gamma_5) l_j] + h.c.$$

Flavour Changing Charged Current





THE XXI CENTURY PERIODIC TABLE



THE STRUCTURE OF THE SM

Gauge bosons

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Fermions

$$\begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix} \; ; \; U_R^i, D_R^i \; ; \; \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \; ; \; e_R^i \; ; \; i = 1, 2, 3 \; generations$$

- They come in 3 generations of Weyl bi-spinors. Note there are NO right-handed neutrinos.

Scalars

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \; ; \; < H^0 > = \; 170 \; GeV$$

- The neutral component of the Higgs $SU(2)_L$ doublet H^0 gets a vev and breaks the symmetry down to $U(1)_{em}$. $W^{\pm}, Z^0$ become massive.

- Quarks and leptons get their masses from Yukawa couplings to the Higgs doublet :

CHIRALITY

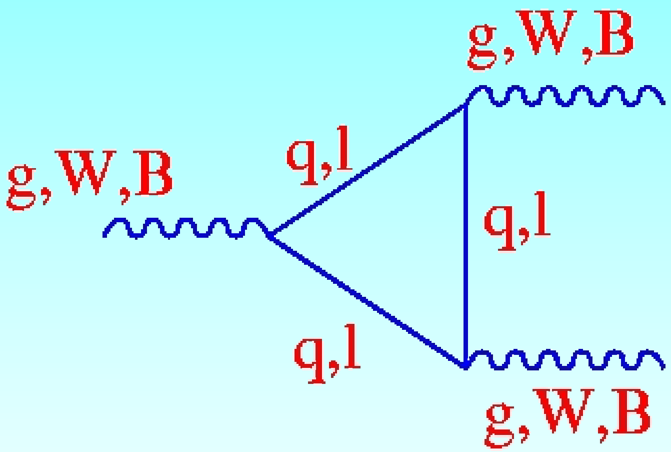
- This is perhaps the most remarkable property of the SM.
- Each generation of quark and leptons have the following quantum numbers under SM gauge group:

Fermion	$SU(3)$	$SU(2)_L$	$U(1)_Y$
$Q_L = (U, D)_L$	3	2	1/6
$U_R = U_L^c$	$\bar{3}$	1	-2/3
$D_R = D_L^c$	$\bar{3}$	1	+1/3
$L = (\nu, E)_L$	1	2	-1/2
$E_R = E_L^c$	1	1	+1

- The remarkable point is that left- and right-handed components of the same Dirac spinor have DIFFERENT quantum numbers under the SM group.
- That property is called CHIRALITY and corresponds to the V-A structure of weak interactions.
- (Mathematically this corresponds to the statement that the quarks and leptons live in a complex representation of the SM group).
- Note the peculiar values of hypercharges. They may be understood from *anomaly cancellation*.

CHIRALITY AND ANOMALIES

- Gauge theories with CHIRAL FERMIONS like the SM are typically sick due to one-loop inconsistencies,the ANOMALIES.
- They come from divergent triangle graphs which give rise to a breakdown of unitarity.....



- ....unless they cancel. This happens for certain choices of quantum numbers for the fermions.
- That is the case of the SM. If the hypercharge assignments are given PRECISELY as in the SM, anomalies cancel.

SOME QUESTIONS BEYOND THE SM

i) Values of couplings, masses and mixings

- Can they be computed in some new underlying theory?

ii) The origin of electroweak symmetry breaking

- Comes from an elementary Higgs? Composite? or?

iii) The fine-tuning problems:

- The cosmological constant puzzle
- The strong-CP problem
- The gauge hierarchy problem

iii) Unification with quantum gravity

- String theory?

i) ABOUT COUPLINGS, MASSES AND MIXINGS

- A first thing to understand would be the different intensity of $SU(3)$, $SU(2)$ and $U(1)$ interactions, i.e. the value of the three gauge coupling constants g_1, g_2, g_3.
- One should add the self-coupling λ of the Higgs field.
- On the other hand one of the most outstanding puzzles of the standard model is the structure of fermion masses and mixing angles.
- The masses of quark and leptons are clearly not random, showing a hierarchical structure.

U-quarks	u	c	t
	0.9-2.9 MeV	530-680 MeV	168-180 GeV
D-quarks	d	s	b
	1.8-5.3 MeV	35-100 MeV	2.8-3.0 GeV
Leptons	e	μ	τ
	0.51 MeV	105.6 MeV	1.777 GeV

Table 1: Masses of quarks and leptons at the M_Z scale (Fritzsch, Xing(2000)).

- Concerning mixing angles, the experimental measurements yield:

$$|V_{CKM}| = \begin{pmatrix} 0.9741-0.9756 & 0.219-0.226 & 0.0025-0.0048 \\ 0.219-0.226 & 0.9732-0.9748 & 0.038-0.044 \\ 0.004-0.014 & 0.037-0.044 & 0.9990-0.9993 \end{pmatrix}. \quad (1)$$

- This is close to a unit matrix with small off-diagonal mixing except for the Cabbibo (12) entry which is somewhat larger.
- Experiments also yield CP-violation which in terms of the Jarlskog invariant J is $J = (3.0 \pm 0.3) \times 10^{-5}$.
- The situation has become even more challenging after the experimental confirmation of neutrino oscillations.
- It has been found for neutrino mass^2 differences from solar ν's, KamLAND ant atmospheric:

$$\Delta m_{12}^2 = 7.1 \times 10^{-5}\, eV^2;\ \Delta m_{23}^2 = 1.3 - 3.0 \times 10^{-3}\, eV^2$$

- The pattern for neutrino mixing seems to be quite different from the CKM case [a]

$$\begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.0-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix}. \qquad (2)$$

- ν-mixing angles are typically large.
- All of these couplings, masses and mixings cannot be understood from just the SM. They are input parameters and their explanation should come from physics BSM.
- Vigorous attempts have been made in the last 25 years in order to understand these questions.
- Particularly attractive are Grand Unified Theories (GUT's). In GUT's one can:
  - Relate the three gauge coupling constants to each other
  - Understand the smallness of neutrino masses
  - Relate masses m_b, m_τ, m_t
  - Understand baryon number generation (e.g. Leptogenesis)

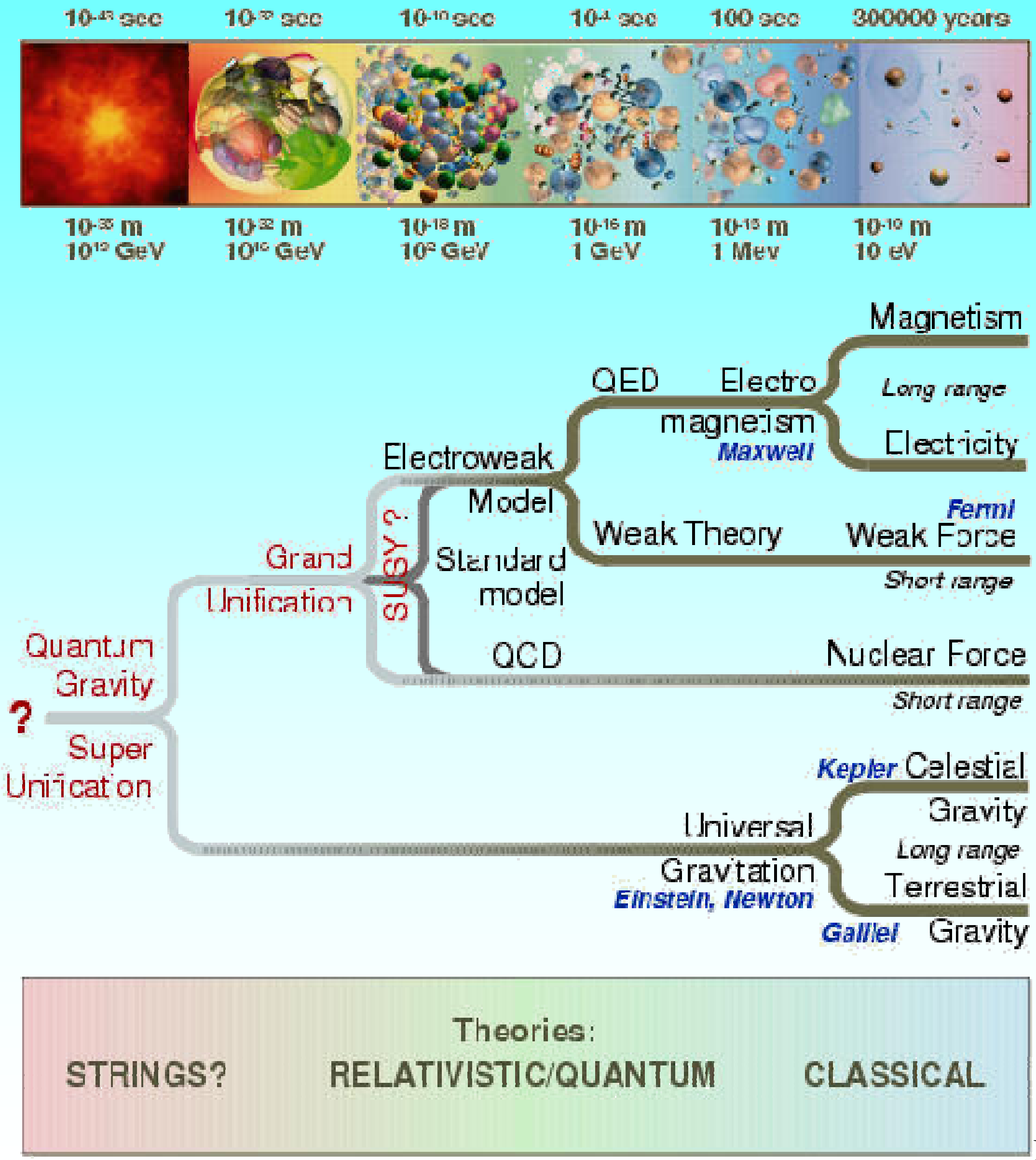
[a] e.g. Gonzalez-Garcia, Peña-Garay (2003)

- On the other hand not equally compelling theories exist to explain the full fermion spectra and mixings. Some attempts include:
  - Horizontal $U(1)$ symmetries (Frogatt-Nielsen)
  - GUT's supplemented with discrete symmetries
  - Specific string compactification models
  - Phenomenological models of extra dimensions *(Split Fermions)*
- I will not discuss much these possibilities but rather concentrate first on Grand Unified Theories.

GRAND UNIFIED THEORIES

Leandar Litov

Interactions unification



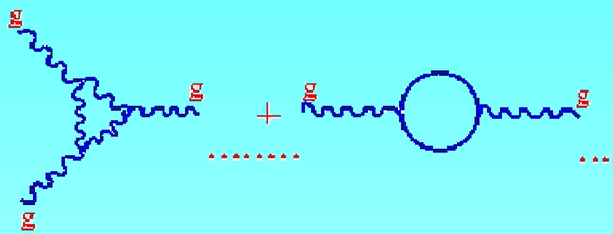
GRAND UNIFIED THEORIES

- In the SM there are FIVE multiplets per generation: $Q_L, U_L^c, D_L^c, L, E_L^c$.
- Gran Unified Theories [a] assume there is an underlying gauge symmetry G larger than $SU(3) \times SU(2) \times U(1) \in$ G.
- Quarks and leptons then unify into a smaller number of multiplets (e.g. two in $SU(5)$, one in $SO(10)$).
- Instead of the three independent gauge couplings $g_{1,2,3}$ of the SM, there is a single one g_{GUT}. This leads to predictions for gauge couplings.
- The unification takes place at a very large scale $\propto 10^{15}$ GeV. Its effects can only be checked indirectly.
- A particularly compelling motivation is the running of the SM gauge couplings

[a] Georgi,Glashow;Pati,Salam (1974)

THE SM GAUGE COUPLINGS RUN

- Due to renormalization effects:



- Renormalization group equations :

$$\mu \frac{\partial}{\partial \mu} \alpha_i(\mu) = \frac{1}{2\pi} b_i \alpha_i^2(\mu)$$

μ = energy scale ; b_i= β-function coefficients.

- For $SU(N)$ gauge theory one has:

$$b_i = -\frac{11}{3} N + \frac{1}{3} n_f + \frac{1}{6} n_s$$

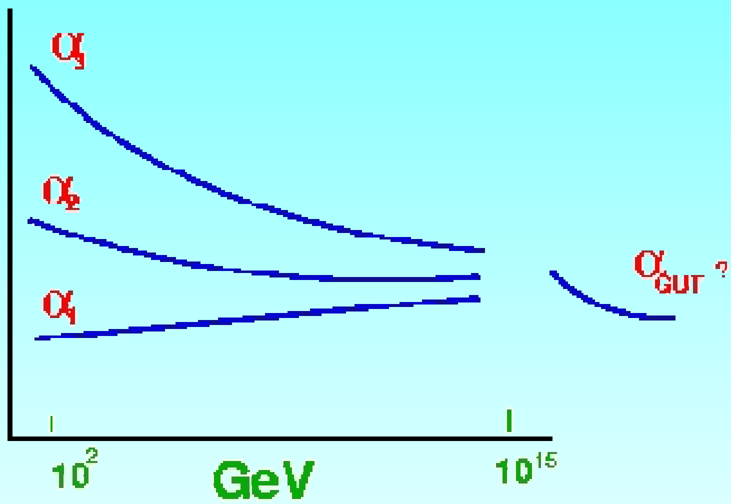
with $n_{f,s}$ = number of fermion(scalar) N-plets.

- For the Standard Model one has:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{gen} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

- Integrating the renormalization group equations [a] (one loop):

$$\frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{2\pi} log(\frac{\mu^2}{q^2}) \ ; \ i = 1,2,3$$



- Suggests the existence of a unified theory at scales of order 10^{15} GeV.

- Simplest Lie groups containing $SU(3) \times SU(2) \times U(1)$ and having chiral fermions: $SU(5)$, $SO(10)$.

[a] Georgi, Quinn, Weinberg (1974)

GRAND UNIFICATION: SU(5)

- 24 GAUGE BOSONS (Adjoint of $SU(5)$)

$$A^{\mu}_{SU(5)} = \begin{pmatrix} & 8\,gluons & & X_1^- & Y_1^- \\ & & & X_2^- & Y_2^- \\ & & & X_3^- & Y_3^- \\ X_1^+ & X_2^+ & X_3^+ & Z^0 & W^- \\ Y_1^+ & Y_2^+ & Y_3^+ & W^+ & \gamma \end{pmatrix}$$

$X_i^{\pm}, Y_i^{\pm}$ very massive, $M_{X,Y} \propto 10^{15}$ GeV.

- FERMIONS: each generation: 10+$\overline{5}$

$$\overline{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix} \; ; \; 10 = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}$$

- One generation of quarks and leptons JUST FIT!
- The 10+$\overline{5}$ combination is ANOMALY FREE

$SU(5)$ Symmetry Breaking

- $SU(5)$ symmetry is broken spontaneously at a large scale by the vacuum expectation value of scalars Φ_a in the adjoint of $SU(5)$:

$$<\Phi_a> = \begin{pmatrix} 2V & 0 & 0 & 0 & \\ 0 & 2V & 0 & 0 & 0 \\ 0 & 0 & 2V & 0 & 0 \\ 0 & 0 & 0 & -3V & 0 \\ 0 & 0 & 0 & 0 & -3V \end{pmatrix}$$

V $\propto 10^{15}$ GeV $\rightarrow M_{X,Y} \propto 10^{15}$

$SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

- Further symmetry breaking $SU(2) \times U(1) \rightarrow U(1)_{em}$ by scalars H_5 in a 5-plet:

$$H_5 = \begin{pmatrix} H_1^c \\ H_2^c \\ H_3^c \\ H_{WS}^{\pm} \\ H_{WS}^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

SOME IMPLICATIONS OF GUT's

1) Charge quantization

- Electromagnetic charge Q_{em} is an $SU(5)$ generator: Trace (Q_{em}) = 0 :

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix} \rightarrow Q_{em} = \begin{pmatrix} Q(d^c) & 0 & 0 & 0 & \\ 0 & Q(d^c) & 0 & 0 & 0 \\ 0 & 0 & Q(d^c) & 0 & 0 \\ 0 & 0 & 0 & Q(e^-) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow Q(d) = \frac{1}{3} Q(e^-)$$

- Explains an important fact of the observed world: charges of the proton and electron should be equal and opposite.

2) Prediction of Weak Mixing Angle

- There is only one gauge coupling $g_5 \longrightarrow \alpha_{em}, \alpha_s$, $sin^2\theta_W$ are RELATED

- $$sin^2\theta_W = \frac{Tr(T_3^2)}{Tr(Q_{em}^2)}$$

$$Q_{em}^2 = \begin{pmatrix} 1/9 & 0 & 0 & 0 & \\ 0 & 1/9 & 0 & 0 & 0 \\ 0 & 0 & 1/9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T_3^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1/4 \end{pmatrix}$$

$\longrightarrow$

$$sin^2\theta_W = \frac{1/2}{12/9} = \frac{3}{8}$$

- That is the value at the GUT scale!
- Below the GUT scale the couplings run differently in a way which is computable using the renormalization group equations

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_{GUT}} + \frac{b_i}{2\pi} log(\frac{M_{GUT}}{M_W}) \; ; \; i = 1,2,3$$

- Combining the three equations one gets:

$$\frac{1}{\alpha_3(M_W)} = \frac{3}{8}(\frac{1}{3\alpha_{em}(M_W)} - \frac{1}{2\pi}(b_1 + b_2 - \frac{8}{3} b_3)\, log(\frac{M_{GUT}}{M_W}))$$

$$sin^2\theta_W(M_W) = \frac{3}{8} + \frac{5\alpha_{em}(M_W)}{16\pi}(b_2 - \frac{3}{5} b_1)\, log(\frac{M_{GUT}}{M_W})$$

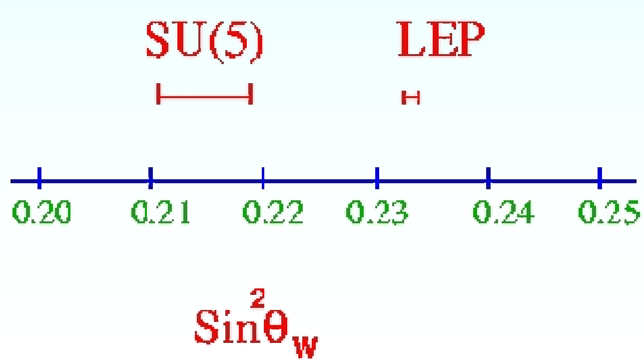
- With e.g. the first of those equations one can compute:

$$M_{GUT} = 10^{14} - 10^{15} GeV$$

- Then one gets a prediction for the weak angle :

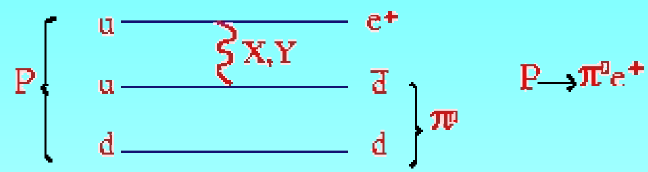
$$sin^2\theta_W(M_W) = 0.214 \pm 0.004(3)$$

- This is ruled out by LEP results:



3) Baryon Number Violation

- Since quarks and leptons live inside same multiplets, X,Y massive gauge bosons can mediate baryon number violating transitions. E.g. $SU(5)$:



- On dimensional grounds, since amplitude goes like $1/M_X^2$, proton time-life goes like:

$$\tau_P \propto \frac{M_X^4}{m_P^5}$$

A more sophisticated theoretical computation gives

$$\tau(P \to e^+\pi^0) = 4 \times 10^{29 \pm .7} years$$

Compared to experimental (Super Kamiokande) bounds:

$$\tau(P \to e^+\pi^0) > 5.4 \times 10^{33} years$$

- Thus (minimal) $SU(5)$ is excluded

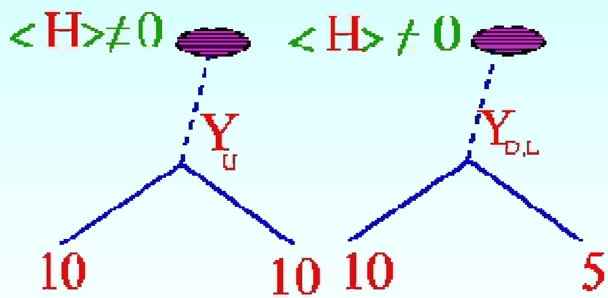
4) Relationships among fermion masses

- Recall that in the SM the fermion masses arise from three independent sets of Yukawa couplings of the fermions to the Higgs doublet H_{WS}:

$$L_{Yuk} = Y_U^{ij}\overline{Q}_L^i U_R^j H^* + Y_D^{ij}\overline{Q}_L^i D_R^j H + Y_L^{ij}\overline{L}^i E_R^j H + h.c.$$

- In e.g. $SU(5)$ the number of independent Yukawa couplings is reduced:

$$L_{Yuk}^{SU(5)} = Y_U^{ij}\overline{\psi}_{10}^i \psi_{10}^j H_5^* + Y_{D,L}^{ij}\overline{\psi}_{\bar{5}}^i \psi_{10}^j H_{\bar{5}} - h.c.$$

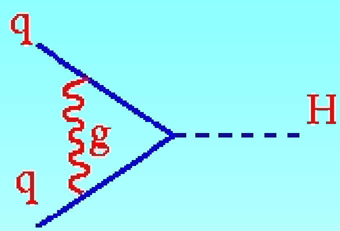


- Thus at the GUT scale one has a relationship between the Yukawas of charged leptons and D-quarks: $Y_D^{ij} = Y_L^{ij}$

- In particular this means [a]

$$Y_b(M_X) = Y_\tau(M_X)\,;\; Y_s(M_X) = Y_\mu(M_X)\,;\; Y_d(M_X) = Y_e(M_X)$$

- However the Yukawa couplings run with the energy. In particular $SU(3)$ loop corrections enhance the quark Yukawas compared to the charged leptons as one goes down in energies.



- The leading correction yields e.g. for the 3-d generation:

$$\frac{m_b(M_W)}{m_\tau(M_W)} = \left(\frac{\alpha_3(M_W)}{\alpha_3(M_X)}\right)^{-\gamma/2b_3} \simeq 3$$

with γ the *anomalous dimension* coefficient.

- This result is quite good for the 3-d generation. However, the identity fails for the first and second generations

- More complicated GUT models involving several other Higgs multiplets beyond $H_{\bar{5}}$ are able to fit the data.

[a] Buras et al. (1978)